

$$1) \int \sin^6 x$$

$$\Rightarrow \int \sin^6 x \, dx$$

$$\Rightarrow (\sin^2 x)^2 (\sin x)$$

$$= \int \left(\frac{1 - \cos 2x}{2} \right)^2 \left(\frac{1 - \cos 2x}{2} \right)$$

$$\rightarrow \frac{1}{8} \int (1 - 2\cos 2x + \cos^2 2x) (1 - \cos 2x)$$

$$\Rightarrow \frac{1}{8} \int \left(1 - 2\cos 2x + \frac{1 + \cos 4x}{2} \right) (1 - \cos 2x)$$

$$= \frac{1}{16} \int (1 - \cos 2x + 1 + \cos 4x) (1 - \cos 2x)$$

$$\Rightarrow \frac{1}{16} \int (3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x)$$

$$= \frac{1}{16} \int (3 - 7\cos 2x + \cos 4x + 2 \times 2\cos^2 2x - \frac{1}{2} \times 2\cos 4x \cos 2x)$$

$$= \frac{1}{32} \int (10 - 15\cos 2x + \cos 4x - \cos 6x) \, dx$$

$$= \frac{1}{32} \left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6} \right] + c$$

$$= \frac{5x}{8} - \frac{15\sin 2x}{64} + \frac{3\sin 4x}{64} - \frac{\sin 6x}{192} + c //$$

$$= \frac{1}{32} \int (6 - 14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x)$$

$$= \frac{1}{32} \int (10 - 15\cos 2x + 6\cos 4x - \cos 6x)$$

$$\Rightarrow \frac{1}{32} \left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6} \right] + c$$

$$\Rightarrow \frac{5x}{16} - \frac{15\sin 2x}{64} + \frac{3\sin 4x}{64} - \frac{\sin 6x}{192} + c //$$

$$2) \int \cos^4 x \sin^3 x$$

$$\Rightarrow u = \cos x$$

$$\Rightarrow \frac{du}{dx} = -\sin x \Rightarrow dx = -\frac{du}{\sin x}$$

$$\sin^2 x = (1 - \cos^2 x)$$

$$\Rightarrow \int \sin x \cdot \sin^2 x \cdot u^4 \cdot \frac{-du}{\sin x}$$

$$\Rightarrow \int (1 - \cos^2 x) \cdot u^4 \cdot du \Rightarrow \int (1 - u^2) \cdot u^4 \cdot du$$

$$\Rightarrow \int (u^4 - u^6) \cdot du$$

$$\Rightarrow - \left[\frac{u^5}{5} - \frac{u^7}{7} \right] + c$$

$$\Rightarrow \frac{u^7}{7} - \frac{u^5}{5} + c =$$

$$\Rightarrow \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + c //$$

$$\Rightarrow \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c //$$

$$3) \int \cos x \sin^3 x$$

$$; u = \sin x$$

$$\frac{du}{dx} = \cos x \Rightarrow dx = \frac{du}{\cos x}$$

$$\int \cos x \cdot u^3 \cdot \frac{du}{\cos x}$$

$$\Rightarrow \int u^3 \cdot du = \frac{u^4}{4} + c$$

$$\Rightarrow \frac{\sin^4 x}{4} + c //$$