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(1)

$\sin^6 x$

$\int \sin^6 x\,dx$

$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$

$= \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1}{8}\left(1-2\cos 2x+\cos^2 2x\right)(1-\cos 2x)$

$= \frac{1}{8}\left(1-2\cos 2x+\frac{1+\cos 4x}{2}\right)(-\cos 2x+1)$

$= \frac{1}{16}(3-4\cos 2x+\cos 4x)(1-\cos 2x)$

$= \frac{1}{16}[3-4\cos 2x+\cos 4x-3\cos 2x+4\cos^2 2x-\cos 4x\cos 2x)$

$\frac{1}{16}[3-7\cos 2x+\cos 4x+2\times 2\cos^2 2x$

$-\frac{1}{2}\cdot 2\cos 4x\cos 2x$

$\frac{1}{16}[3-7\cos 2x+\cos 4x+2(1+\cos 4x$

$-\frac{1}{2}(\cos 6x+\cos 2x]$

$\frac{1}{16}[3-7\cos 2x+\cos 4x+2+2\cos 4x-$

$\frac{1}{2}(\cos 6x+\cos 2x]$

$$\frac{1}{32}[6-14\cos 2x + 2\cos 4x + 4 + 4\cos 4x - \cos 6x - \cos 2x]$$

$$\frac{1}{32}[10-15\cos 2x + 6\cos 4x - \cos 6x$$

$$\frac{1}{32}\int[10-15\cos 2x + 6\cos 4x - \cos 6x\,dx]$$

$$= \frac{1}{32}\left[10x - \frac{15\sin 2x}{2} + \frac{6\sin 4x}{4} - \frac{\sin 6x}{6}\right] + c$$

$$= \frac{1}{192}[60x - 45\sin 2x + 9\sin 4x - \sin 6x] + c$$

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$\cos^4 x \sin^3 x$

$\sin^3 x \cos^4 x\,dx = (1-\cos^2 x)\cos^4 x \sin x$

$= \cos^4 x \sin x - \cos^6 x \sin x$

$\int(\cos^4 x \sin x - \cos^6 x \sin x)\,dx$

$= -\frac{1}{5}\cos^5 x + \frac{1}{7}\cos^7 x + c$

(3) $\cos x \sin^3 x$

$\int \sin^3 x \cos x \, dx$

let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

Therefore

$= \int u^3 \, du$

$= \frac{u^{3+1}}{3+1} + C = \frac{u^4}{4} + c$

$u = \sin x$

$\int \sin^3 x \cos x \, dx = \frac{\sin^4 x}{4} + c$