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MATRIC NO; 19/MHS01/265

COURSE; MATH 104

(1) $\int \sin^6 x \, dx$

Also the same as $\int (\sin^2 x)^3 = \int \left[\frac{1}{2}(1 - \cos x)\right] dx$

$\frac{1}{8}\int (1-\cos 2x)^3 \, dx = \frac{1}{8}\int (1 - 3\cos 2x + 3\cos^2 x - \cos^3 2x)\, dx$

$\frac{1}{8}\left[\left(x - \frac{3}{2}\sin 2x\right) + \int 3\cos^2 2x - \int \cos^3 2x \, dx\right]$

$= \frac{1}{8}\left[\left(x - \frac{3}{2}\sin 2x + \frac{3x}{2} + \frac{3}{8}\sin 4x\right) - \int \cos^3 2x \, dx\right]$

To find the $\int \cos^3 2x \, dx$

$\int \cos^3 2x \, dx = \int \cos 2x (\cos^2 2x)\, dx = \int \cos 2x (1 - \sin^2 2x)\, dx$

let $u = \sin 2x$

$du = -2\cos 2x \, dx$

$-du/2 = \cos 2x \, dx$

$\int \cos 2x (1 - \sin^2 x) = \int \frac{-du}{2} \cdot (1-u^2) = \frac{1}{2}\int du\,(u^2 - 1) =$

$\frac{1}{2}\left(\frac{u^2}{3} - u\right) = \frac{u^3}{6} - \frac{u}{2} = \frac{(\sin 2x)^3}{6} - \frac{\sin 2x}{2}$

$\int \sin^6 x \, dx = \frac{1}{8}\left[x - \frac{3}{2}\sin 2x + \frac{3x}{2} + \frac{3}{8}\sin 4x - \left(\frac{\sin^3 2x}{6} - \frac{\sin 2x}{2}\right)\right] + C$

$\int \sin^6 x \, dx = \frac{5x}{16} - \frac{3}{2}\sin 2x + \frac{3}{8}\sin 4x - \left(\frac{\sin^3 2x}{6} - \frac{\sin 2x}{2}\right)\Bigg]$

$\int \sin^6 x \, dx = 5x/16 - 3/2 \sin^2 x + 3/8 \sin 4x - 1/48 \sin^3 2x + 1/16 \sin 2x + C$

$\int \sin^6 x \, dx = \frac{5x}{16} - \frac{23}{16}\sin 2x + \frac{3}{8}\sin 4x - \frac{1}{48}\sin^3 + C$

2) $\int \cos^4 x \sin^3 x \, dx$

$u = \cos x$

$du/dx = -\sin x$, $dx = -du/\sin x$

Recall $\sin^2 x = 1 - \cos^2 x$

$\therefore \int \cos^4 x \sin^3 x \, dx = \int u^4 \sin x (1 - \cos^2 x) \; -du/\sin x$

$\int (u^2 - 1) \cdot u^4 \, du = \int (u^6 - u^4) \, du = \frac{u^7}{7} - \frac{u^5}{5} + c$

$\therefore \int \cos^4 x \sin^3 x \, dx = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + c$

3. $\int \cos x \sin^3 x \, dx$

Let $u = \sin x$, $du/dx = \cos x$

$du = \cos x \, dx$

$\therefore \int \cos x \sin^3 x \, du = \int u^3 du = \frac{u^4}{4} + c = \frac{(\sin x)^4}{4} + c$

$\therefore \int \cos x \sin^3 x \, dx = \frac{(\sin x)^4}{4} + c$