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MBBS
MATHS 104

1) $\sin^6 x \, dx$

$$\sin^6 x = (\sin^2 x)^2 (\sin^2 x)$$

$$= \left(\frac{1 - \cos 2x}{2}\right)^2 \left(\frac{1 - \cos 2x}{2}\right)$$

$$= \frac{1}{8}\left(1 - 2\cos 2x + 1 + \cos 4x\right)(1 - \cos 2x)$$

$$= \frac{1}{8}\left(1 - 2\cos 2x + \frac{1 + \cos 4x}{2}\right)(1 - \cos 2x)$$

$$\frac{1}{16}\left(2 - 4\cos 2x + 1 + \cos 4x\right)(1 - \cos 2x)$$

$$= \frac{1}{16}\left(3 - 4\cos 2x + \cos 4x\right)(1 - \cos 2x$$

$$= \frac{1}{16}\left(3 - 4\cos 2x + \cos 4x - 3\cos 2x + 4\cos^2 2x - \cos 4x \cos 2x\right)$$

$$= \frac{1}{16}\left(3 - 7\cos 2x + \cos 4x + (2(2\cos^2 2x) - \tfrac{1}{2}\, 2\cos 4x \cos 2x\right)$$

$$= \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2(1 + \cos 4x) - \frac{1}{2}(\cos 6x + \cos 2x)\right]$$

$$= \frac{1}{16}\left[3 - 7\cos 2x + \cos 4x + 2 + 2\cos 4x - \frac{1}{2}(\cos 6x + \cos 2x)\right]$$

$$= \frac{1}{32}\left[6 - 14\cos 2x + 2\cos 4x + 4 + \cos 4x - \cos 6x - \cos 2x\right]$$

$$= \frac{1}{32}\left[10 - 15\cos 2x + 6\cos 4x - \cos 6x\right]$$

Let $\sin^6 x = R$

$$R = \frac{1}{32}\left(10x - \frac{15 \sin 2x}{2} + \frac{6\cos 4x}{4} - \frac{\cos 6x}{6}\right) + C$$

$$\int \sin^6 x = \frac{10x}{32} - \frac{15\sin 2x}{64} + \frac{6\cos 4x}{128} - \frac{\cos 6x}{192} + C$$

2) $\int \cos^4 x \sin^3 x \, dx$

$u = \cos x$

$= \frac{du}{dx} = -\sin x \quad dx = \frac{-du}{\sin x}$

recall that $\sin^2(x) + \cos^2(x) = 1$

$\sin^2(x) = 1 - \cos^2(x)$

$= \sin^2 x . u^4 \, du$

$(1 - \cos^2 x) . u^4 \, du$

$(u^2 - 1) u^4 \, du$

$u^6 - u^4) \, du$

$\frac{u^7}{7} - \frac{u^5}{5}$

Substitute $u = \cos x$

$\frac{\cos^7 x}{7} - \frac{\cos^5 x}{5}$

$\cos^4 x \sin^3 x \, dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5}$

(3) $\cos x \sin 3x \, dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x \quad dx = \frac{-du}{\sin x}$

recall $\sin^2 x = 1 - \cos^2 x$

$\int - \cos x - 4 \cos^3(x) \sin x \, dx$

$\int u - 4u^3 . \sin x \, dx$

$\int (4u^3 - u) \, du$

$\frac{u^2}{2} - u^4$

Substitute $u = \cos x$

$\frac{\cos^2 x}{2} - \cos^4 x + C$

$\therefore \int \cos x \sin 3x \, dx = \frac{\cos^2 x}{2} - \cos^4 x + C$