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DEPARTMENT: MBBS

1. $\sin^6 x$

$\sin^2 x = 1-\cos^2 x$

$\int (1-\cos^2 x)^3 \, dx$

$\int (1-\cos^2 x)^2 (1-\cos^2 x)\, dx$

$\int (1-2\cos^2 x+\cos^4 x)(1-\cos^2 x)\, dx$

$\int (1-3\cos^2 x+3\cos^4 x-\cos^6 x)\, dx$

Using $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$$\int 1-3\left(\frac{1}{2}(1+\cos 2x)\right)+3\left(\frac{1}{4}(1+\cos 2x)^2\right)-\left(\frac{1}{8}(1+\cos 2x)^3\right) dx$$

$$\int 1-\frac{3}{2}(1+\cos 2x)+\frac{3}{4}(1+\cos 2x)^2-\frac{1}{8}(1+\cos 2x)^3 \, dx$$

$$\int 1-\frac{3}{2}-\frac{3}{2}\cos 2x+\frac{3}{4}+\frac{3}{2}\cos 2x+\frac{3}{4}\cos^2 2x-\frac{1}{8}-\frac{3}{8}\cos 2x-\frac{3}{8}\cos^2 2x-\frac{1}{8}\cos^3 2x \, dx$$

$$\int -\frac{1}{2}\cancel{-\frac{3}{2}\cos 2x}+\frac{3}{4}+\cancel{\frac{3}{2}\cos 2x}+\frac{3}{4}\cos^2 2x-\frac{1}{8}-\frac{3}{8}\cos 2x-\frac{3}{8}\cos^2 2x-\frac{1}{8}\cos^3 2x \, dx$$

$$\int \frac{1}{8}+\frac{3}{4}\cos^2 2x-\frac{3}{8}\cos 2x-\frac{3}{8}\cos^2 2x-\frac{1}{8}\cos^3 2x \, dx$$

$$\int \frac{1}{8}+\frac{3}{4}\frac{(1+\cos 4x)}{2}-\frac{3}{8}\cos 2x-\frac{3}{8}\frac{(1+\cos 4x)}{2}-\frac{1}{8}(\cos^3 2x-\cos 2x)\, dx$$

$$\int \frac{1}{8}+\frac{3}{8}+\frac{3}{8}\cos 4x-\frac{3}{8}\cos 2x-\frac{3}{16}-\frac{3}{16}\cos 4x-\frac{1}{8}(\cos^2 2x-\cos 2x)\, dx$$

$$\int \frac{5}{16}+\frac{3}{8}\cos 4x-\frac{5}{8}\cos 2x-\frac{3}{16}\cos 4x-\frac{1}{8}(\cos^2 2x-\cos 2x)\, dx$$

$$\int \frac{5}{16}+\frac{3}{8}\cos 4x-\frac{3}{8}\cos 2x-\frac{3}{16}\cos 4x-\frac{1}{8}(1-\sin 2x)(\cos 2x)\, dx$$

$$\int \frac{5}{16}+\frac{3}{8}\cos 4x-\frac{3}{8}\cos 2x-\frac{3}{16}\cos 4x-\frac{1}{8}\int(1-\sin 3x)(\cos 2x)\, dx$$

$$\int \frac{5}{16}+\frac{3}{6}\cos 4x-\frac{3}{8}\cos 2x-\frac{3}{16}\cos 4x$$

$$= \frac{5x}{16} + \frac{3\sin 4x}{32} - \frac{3}{16}\sin 2x - \frac{3}{64}\sin 4x$$

Finding $\int (1-\sin^2 2x)(\cos 2x)$

let $y = \sin 2x$

$dy/dx = 2\cos 2x$

$\frac{1}{2} dy = \cos 2x\, dx$

$\frac{1}{2}\int 1 - y^2\, dy$

$= \frac{1}{2}\left(y - \frac{y^3}{3}\right)$

$= \frac{y}{2} - \frac{y^5}{6}$

$= \frac{\sin 2x}{2} - \frac{\sin^3 2x}{6}$

Add everything

$$= \frac{5}{16}x + \frac{3}{32}\sin 4x - \frac{2}{16}\sin 2x - \frac{3}{64}\sin 4x - \frac{1}{8}\left(\frac{\sin 2x}{2} - \frac{\sin^3 2x}{6}\right) + C$$

$$= \frac{5}{16}x + \frac{3}{32}\sin 4x - \frac{3}{16}\sin^3 2x - \frac{3}{64}\sin 4x - \frac{\sin 2x}{16} + \frac{\sin^3 2x}{48} + C$$

$$\int \sin^6 x = \frac{60x + 18\sin 4x - 36\sin 2x - 9\sin 4x - 12\sin 2x + 4\sin^3 2x}{192} + C$$

$$\int \sin^6 x = \frac{60x + 9\sin 4x - 48\sin 2x + 4\sin^3 x}{192} + C$$

2. $\int \cos^4 x \sin^3 x\, dx$

Let $u = \cos x$

$\frac{du}{dx} = \sin x \quad dx = \frac{-du}{\sin x}$

$$\int u^4 \cdot \sin^3 x \cdot \frac{-du}{\sin x}$$

$- \int u^4 \sin^2 x\, du$

$\sin^2 x = 1 - \cos^2 x$

$- \int u^4 (1 - u^2)\, du$

$- \int u^4 - u^6\, du$

$= \int u^6 - u^4 \, du$

$= \frac{u^7}{7} - \frac{u^5}{5} + C$

$\int \cos^4 x \sin^3 x \, dx = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

3. $\int \cos x \sin^3 x \, dx$

Let $u = \cos x$

$\frac{du}{dx} = \frac{-du}{\sin x}$

$\int u \cdot \sin^3 x \cdot \frac{-du}{\sin x}$

$- \int u \sin^2 x \, du$

$- \int u(1-u^2) du$

$- \int u - u^3 du$

$\int u^3 - u \, du$

$= \frac{u^4}{4} - \frac{u^2}{2} + C$

$\int \cos x \sin^3 x \, dx = \frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + C$