Revision Questions

1. Differentiate between computability and complexity based on their objective

computability theory and complexity theory are both under the branch of 'theory of computation' However they differ in the following ways!

Computability theory is concerned with the question of whether a problem can be solved on a computer at all or by algorithms (by turing machine). It addresses four main questions

- what problems can Turing machines solve?
- what other systems are equivalent to turing machines?
- what problems require more powerful machines
- what problems can be solved by less powerful machines?

While Complexity is concerned with the scalability of computational problems and algorithms, it places practical limits on what computers can and cannot reasonably accomplish it deals with the resources (i.e time and space) required during computation to solve a given problem

2. With a given example describe Complexity theory and Computational theory

Example of Computation theory :-

The Entscheidungs problem (decision problem): given a statement in first-order predicate calculus, decide whether it is universally valid. Church and Turing independently proved this is undecidable

Example of Complexity theory :

Mowing grass has linear complexity because it takes double the time to mow double the area

(occwiki.org, 2009)

3. Define the following with a given example

(i) A set :- it is a well defined collection of a particular objects e.g $A = \{2, 4, 6, 8, 10, ...\}$

A is a set of even numbers

(ii) Power set : it is the set of all subsets of a ~~set~~ including the empty set and itself

$R = \{a, b, c\}$

$P(R)$ [powerset of R] $= \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

(iii) Members of a set = these are the objects that make up a set i.e $A = \{2, 4, 7\}$

2, 4, 7 are members of A i.e $2, 4, 7 \in A$

iv) Subset! ~~A subset~~ set A is a subset of a set B if every element in A is also a member of B.

$A \subseteq B$ if $B = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{2, 4, 6\}$ $\therefore$ $A \subseteq B$

v) Proper Subset! A is a subset (proper subset) of a set B if ~~a set~~ A is not equal to B but all the elements in A are in B i.e $\subset$

$B = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{2, 3\}$ $\therefore$ $A \subset B$

vi) Infinite set! this is a set of objects that does not have a definite or last term in particular the set goes on till infinite

e.g the set of all even numbers

i.e $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, \ldots\}$

vii) Finite set! this is a set that has a definite end, it has a limited amount of terms and does not span till infinity

i.e Set of even numbers before 10 and after 0

$B = \{2, 4, 6, 8\}$

viii) Unordered Pair! is a set of form $\{a, b\}$ having those elements with no particular relation between them

eg $\{2, 2\} = \{2, 2\}$

ix) Union of set! Union refers to the ~~total~~ all the elements present in the sets that have been 'union'ed i.e $\cup$

e.g. $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

x) intersection of set: refers to all the elements that two sets have in common i.e $\cap$

e.g $A = \{2, 4, 6, 8\}$, $B = \{1, 2, 3, 4\}$

$A \cap B = \{2, 4\}$

xi) complement of a set: is the set of all the elements which are not present in a particular set but are in the universal set 'U' i.e A'

e.g $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 3, 5, 7, 9\}$, $\therefore A' = \{2, 4, 6, 8, 10\}$

xii) Difference of a set: Difference of set A from set B (i.e $A - B$) refers to a set containing all the elements in A that are not in B

e.g $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8, 9\}$

$A - B = \{1, 3, 5\}$

xiii) Symmetric difference of a set ~~refers~~: Symmetric difference of A and B i.e $A \oplus B$ refers to all the elements that the sets DO NOT have in common i.e all the things in A not in B and all the things in B not in A

e.g $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8, 9\}$

$A \oplus B = \{1, 3, 5, 8, 9\}$

4.) Consider the universal set $U = \{1, 3, 5, \ldots, 9\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

$A = \{2, 3, 5, 6, 9\}$, $B = \{0, 2, 4, 8, 16, 20\}$

$C = \{1, 3, 5, 7, 9, 11, \ldots\}$ and

$D = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

Find

(i) $B \cap C = \{\emptyset$ or $\{\}$

(ii) $A \cap D = \cancel{\{1, 2, 3, 4, 5, 6, 7\}}, \{2, 3, 5, 6, 9\}$

(iii) $A \cap C = \{3, 5, 9\}$

(iv) $A \cup B = \{0, 2, 3, 4, 5, 6, 8, 9, 16, 20\}$

$B \cup D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 16, 20\}$

$C \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots\}$

$B^c = \{1, 3, 5, 6, 7, 9\}$

$D^c = \{\}$ or $\emptyset$

$C^c = \{2, 4, 6, 8\}$

$A - B = \{3, 5, 6, 9\}$

$D - C = \{2, 4, 6, 8\}$

$C - B = \{1, 3, 5, 7, 9, \ldots\}$

$\cancel{A \oplus C = \{1, 2, 3, 5, 6, 7, 9, \ldots\}}$.

$\cancel{B \oplus C = \{0, 1, 2, 3, 4, 5, 7}$.

$A \oplus C = \{1, 2, 6, \ldots\}$

$B \oplus C = \{0, 1, 2, 3, 4, 5, 7, 8, 16, 20, \ldots\}$

$C \oplus D = \{2, 4, 6, 8, 11, \ldots\}$

$D \oplus B = \{0, 1, 3, 5, 6, 7, 9, 16, 20\}$

5. Define the followins

(i) Alphabet :- a set that consists of symbols, letters, or forms that concatenate into strings of the language

(ii) Words : strings concatenated from symbols of the alphabet

(iii) Length of a word :- it is the number of letters (elements of the alphabet) that it is composed of

(iv) Substring ! a contiguous sequence of characters within a string

v) Initial segment ! this is a substring beginning with the first letter of it's string

vi) Concatenation of strings ! is the word obtained by writing down the letters of the initial string followed by the other strings. It consists of all strings of form vw, where v is a string from L_1 and w is a string from L_2

vii) Language is a set of words over an alphabet is a collection of words belonging to or derived from an alphabet

6. Suppose A = {a, c, e, i, o, n, t}

with strings V = concatenation and

U = catetioncona

(i) Find the length of word in V: |V| = 13

iii) Reverse string U (U) = noitanetacnoc

ii) Concatenate V and U:

VU = concatonationcatetioncona

iv) Find substrings of string m y m = cacontatione

Σ^0 = {λ or ε}

Σ^1 = {c, a, c, o, n, t, i, e}

Σ^2 = {ca, co, ac, on, nt, at, ti, io, on, ne}

Σ^3 = {cac, aco, con, ont, nta, tat, ati, tio, ion, one}

Σ^4 = {caco, acon, cont, onta, ntat, tati, atio, tion, ione}

Σ^5 = {cacon, acont, conta, ontat, ntati, tatio, ation, tione}

Σ^6 = {cacont, aconta, contat, ontati, ntatio, tation, atione}

Σ^7 = {caconta, acontat, contati, ontatio, ntation, tatione}

Σ^8 = {cacontat, acontati, contatio, ontation, ntatione}

Σ^9 = {cacontati, acontatio, contation, ontation

Σ^{10} = {cacontatio, acontation, contatione}

Σ^{11} = {cacontation, acontatione}

Σ^{12} = {cacontatione}

Substring of {n = Cacontation}

= {λ, c, a, c, o, n, t, i, e, ca, co, ac, on, nt, at, ti, io, on, ne, cac, aco, con, ont, nta, tat, ati, tio, ion, one, caco, acon, cont, onta, ntat, tati, atio, tion, ione, cacon, acont, conta, ontat, ntati, tatio, ation, tione, cacont, aconta, contat, ontati, ntatio, tation, atione, caconta, acontat, contati, ontatio, ntation, tatione, ~~coc~~ ontat, acontatio, contatio, ontation, ntatione, cacontati, acontatio, contation, ontatione, cacontatio, acontation, contatione, cacontation, acontatione, cacontatione}

7) consider the given word S = abracaba find ~~the initial segment~~ Substring & initial segm.

(A) S $\Sigma^0 = \{\lambda \text{ or } \varepsilon\}$

$\Sigma^1 = \{a, b, r, c\}$

$\Sigma^2 = \{ab, br, ra, ac, ca, ba\}$

$\Sigma^3 = \{abr, bra, rac, aca, cab, aba\}$

$\Sigma^4 = \{abra, brac, raca, acab, caba\}$

$\Sigma^5 = \{abrac, braca, racab, acaba\}$

$\Sigma^6 = \{abraca, bracab, racaba\}$

$\Sigma^7 = \{abracab, bracaba\}$

$\Sigma^8 = \{abracaba\}$

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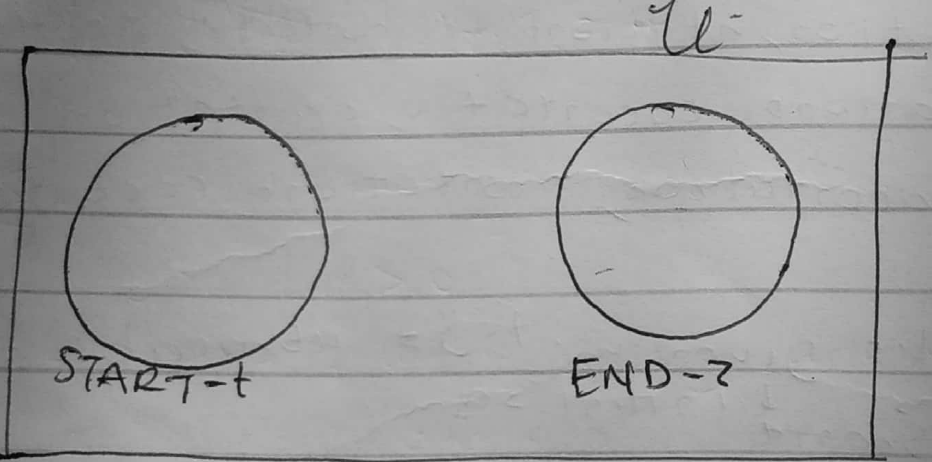
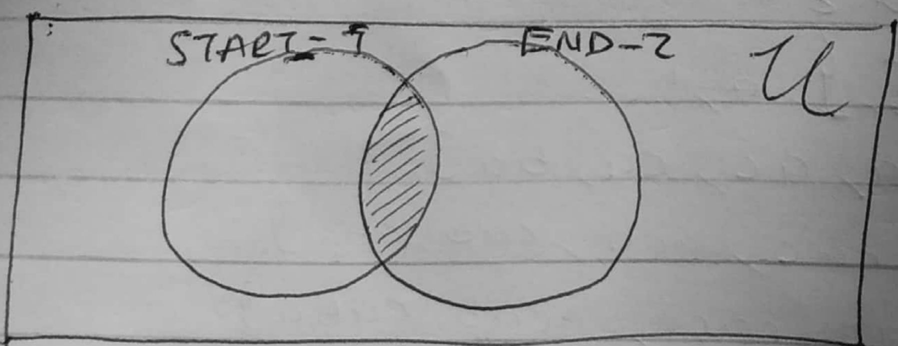
i) Substring of S = {λ, a, b, r, c, ab, br~~a~~, ra, ac, ca, ba, abr, bra, rac, aca, cab, aba, abra, brac, raca, acab, caba, abrac, braca, racab, acaba, abraca, bracab, racaba, abracab, bracaba, abracaba}

ii) Initial segment of S = {a, ab, abr, abra, abrac, abraca, abracab, abracaba}

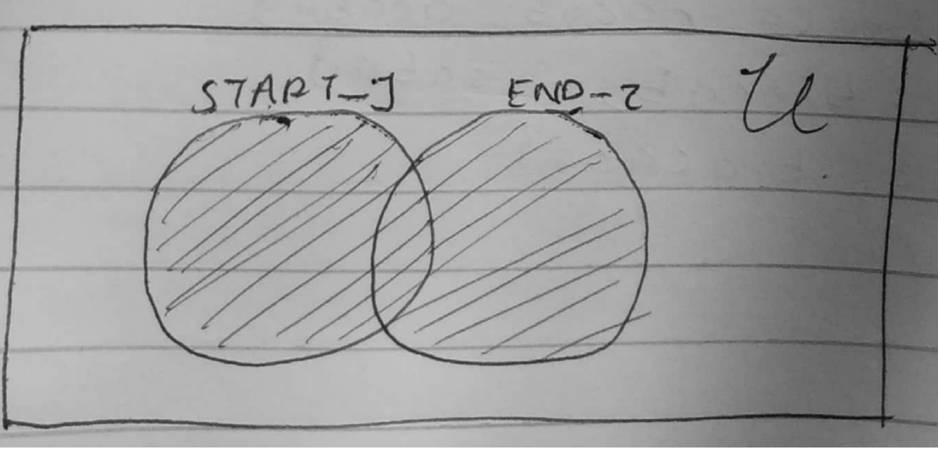
8. (i)



(ii)



(iii)



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9. Σ^*, $L \subseteq \Sigma^*$ and $\Sigma = \{0,1\}$. Find the language that generates

(i) set of all strings contains 11 in it.

~~$L = \{0^n 1^n 0^n$~~ ~~$L = \{0 1^m 0$~~

(i) $L = \{0^n 1^m 0^n \mid n \geq 0,\ m \geq 2\}$

(ii) set of all strings starting with 0 and ending with 1

~~$L = \{0^n 1^m \mid n > 0,$~~

$L = \{0 0^n 1^m 1 \mid n \geq 0,\ m \geq 0\}$

(iii) set of all strings ~~[illegible]~~ with length 2

$L = \{00, 01, 10, 11\}$

$L = \{0^n 1^m \mid \text{where } n+m = 2\}$

(iv) set of all strings ending with '10'

$L = \{0^n 1^m 10 \mid n \geq 0,\ m \geq 0\}$

10) If A denotes alphabet, A^* is set of strings over A

Find the possible alphabet for the following languages

(i) $L = \{en, nouh, ugh\}$

$A = \{e, n, o, u, g, h\}$

(ii) $L = \{bear, rear, 2200\}$

$A = \{a, b, e, r, 0, 2\}$

(iii) the language of all binary strings

$A = \{1, 0\}$.

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11) Given the following set of strings chosen from A^*, $L \subseteq A^*$ and $A = \{0, P\}$. Find the lang usage that search

(i) Set of all strings ending with P

$L = \{P^m O^n P^m \mid n \geq 0, m > 0\}$

(ii) Set of all strings with equal no of O's and P's

$L = \{(P^n O' P O)^n \mid n > 0\}$

(iii) Set of all strings starting with P, ending with O

$L = \{P O^n P^m O \mid n \geq 0, m \geq 0\}$

(iv) Set of all strings with length 4

$L = \{P^n O^m \mid n+m = 4\}$

12) $\Sigma = \{a, b\}$

(i) $L = \{b^n a \mid n > 0\}$

$\therefore$ L consists of words starting with one or more b and ending with one a

(ii) $L = \{a^n a b^m \mid n \geq 0, m \geq 0\}$

L consists of words starting with zero or more a followed by one a and followed by zero or more b

(iii) $\{b^n b^m a^m \mid n \geq 0, m \geq 0\}$

L consists of words starting with ~~one~~ zero or more b followed by ~~one~~ zero or more b followed by zero or more a with the same amount of b just before it

iv) L={ab, abb, aab, aaa, abbb, . . . }
L consists of one or more a followed by zero or more b

3) A = {a, b}
(i) L = {b, ba, bas, baa, bbb, ...}
L consists of languages starting with one or more b's followed by zero or more a's
(iii) { bab, babb, bbab, bbabb, babbb, bbbab, ..}
L consists of one or more bs followed by one a followed by one or more b
iv) {b, bb, bbb, bbbb, . . . }
L consists of one or more b's
(v) {ab, aabb, aaabbb, ... }
L consists of one or more a followed by one or more b of equal length

14) If L, L_1, L_2 are languages under Σ, define

(i) union: $L_1 \cup L_2 = \{u \in \Sigma^* \mid u \in L_1 \text{ or } u \in L_2\}$

(ii) Intersection: $L_1 \cap L_2 = \{u \in \Sigma^* \mid u \in L_1 \text{ and } u \in L_2\}$

iii) difference: $L_1 - L_2 = \{u \in \Sigma^* \mid u \in L_1 \text{ and } u \notin L_2\}$

iv) Complement: $\overline{L} = \{u \in \Sigma^* \text{ and } u \notin L\}$

v) Positive closure: $L^+ = \bigcup_{i=1}^{\infty} L^i = L^1 \cup L^2 \cup \ldots L^\infty$

vi) star operation: $L^* = \bigcup_{i=0}^{\infty} L^i = L^0 \cup L^1 \cup L^2 \cup \ldots L^\infty$

vii) multiplication: - $L_1 L_2 = \{uv \mid u \in L_1, v \in L_2\}$

viii) Power - $L^n = \{\lambda, L^n, L^{n-1}, \ldots \mid n >= 1\}$

OKESOLA AJIBOLA

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