15) Define regular expression for the following languages

i) Language whose length is atmost 2 : Let $\Sigma = \{a, b\}$

$L = \{\varepsilon, a, b, aa, ab, ba, bb$

$(\varepsilon + a + b)(\varepsilon + a + b)$

ii) Language of even length : Let $\Sigma = \{a, b\}$

$((a+b)(a+b))^*$

Let $\Sigma = \{a, b\}$

iii) language starting and ending with the same letter

$a(a+b)^*a + b(a+b)^*b$

iv) Language starting and ending with different letter

$a(a+b)^*b + b(a+b)^*a$

(6) Regular expression consists of constant which denote sets of strings, and operator symbols which denote operations over these sets. they define regular language

Identities of Regular expression

I_1: $\emptyset + R = R$

I_2 $\emptyset R = R\emptyset + \emptyset$

I_3 $\varepsilon R = R\varepsilon = R$

I_4 $\varepsilon^* = \varepsilon$ and $\emptyset^* = \varepsilon$

I_5 = $R + R = R$

I_6 : $R^* R^* = R^*$

I_7 : $RR^* = R^* R$

I_8 $(R^*)^* = R^*$

I_9 $\varepsilon + RR^* = R^* = \varepsilon + R^* R$

I_{10} $(PQ)^* P = P(QP)^*$

I_{11} $(P+Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$

I_{12} $(P+Q)R = PR + QR$ and $R(P+Q) = RP + RQ$

17) A is non-empty alphabet. define r and L(r)

- The symbol λ and the pair () (empty expression) are regular expressions
- each letter in A is a regular expression
- If r is a regular expression then r^* is a regular expression
- If r_1 and r_2 are regular expressions then $r_1 \cup r_2$ is a regular expression and $r_1 r_2$ is a regular expression

the regular expression r is a special word which ~~uses the~~ letters of A and <u>five other symbols</u>: (,), *, $\cup$, λ and nothing else

18) Let A = {a, b}. Describe each of the following r and L(r)

(i) Let $r = b^*$ = {λ, b, bb, bbb, ...}
thus L(r) consists of all the powers of b including the empty word

ii) $r = (aa)^*$ = {λ, aa, aaaa, aaaaaa ...}
thus L(r) consists of words with even length of a including the empty word

iii $r = (a \cup b)^*$ thus L(r) consist of all the powers of a and b including the empty word. It represents all powers of the union of {a} and {b}

iv) $r = a \cup b^*$, $L = \{a, ab, abb, \ldots\}$
thus $L(r)$ consists of words with a single a and all the powers of b including the empty word of b

v) $r = aa(a \cup b^*)$ $L = \{aaa, aaa, aab, \ldots\}$
thus $L(r)$ consists of words starting with two a's and the union of a and all the powers of b

vi) let $r = (a \cap b^*)$ $L = \{a, ab, abb, abbb \ldots\}$
thus $L(r)$ consist of words with one a and intersect with all the powers of b even the empty word

19) Consider the following languages over $\Sigma = \{a, b\}$
$L_1 = \{b^m a^n \mid m > 0, n > 1\}$, $L_2 = \{a^n b^m a \mid m > 1, n > 0\}$
$L_3 = \{ab^n a \mid n > 0\}$. find the regular expression over $A = \{a, b\}$, Such that $L\ L_i = L(r)\ \forall\ i = 1, 2, 3$.

answer!

1) $L_1 = \{b^m a^n \mid m > 0, n > 1\}$
$r = b^*(aa^*)$

2) $L_2 = \{a^n b^m a \mid m > 1, n > 0\}$
$r = aa^* bbb^* a$

3. $L_3 = \{ab^n \mid n > 0\}$
$r = abb^*$

20) What is a regular set

A regular set is any set represented by a regular expression

21) Describe the following sets by regular expression

i) {110} : this is represented by 1 and 0. It is a string from ~~concatenating~~ {1} and {0} by concatenating 1,1,0

ii) {baab} are represented by b and a. It is a string from both {b} and {a} and is made from concatenating b, a, a, b

iii) {01, 10} is the union of {01} and {10}. Therefore is 01+10

iv) {λ, 10} is the union of epsilon and {10} therefore we have λ+10

v) {abb, a, b, bba} is also represented by abb, a, b, bba and is the union of these.

vi) {λ, a, aa, aaa, ...} is represented by {a}* the regular expression is a*

vii) {1, 11, 111, ...} is represented by 1{1}*, the regular expression is 1(1)*

22) Describe the following set with regular expression

i) L-1 is the set of all strings of a's and b's ending in aa

$r = (a+b)^* aa$

ii) L-2 is the set of all strings of 0's and 1's beginning with 1 and ending with 0

$r = 1(0+1)^* 0$

iii) L-3 is the set of {epsilon, 11, 1111, 111111, ...}

$\{\lambda, 11, 1111, 111111, \ldots\}$

$r = (11)^*$

23) A grammar is a powerful tool for describing and analyzing languages. It is a set of rules by which valid sentences in a language are constructed.

A language L over and alphabet Σ is a collection of strings or words on the alphabet while a grammar is a set of rules that is used for describing or analyzing languages

24) Describe the Sentential derivation of string in grammar use a given example to justify this.

Let $G = (V, T, S, P)$ be a grammar. Then the set
$L(G) = \{ w \in T^* : S \overset{*}{\Rightarrow} w \}$ is a language generated by G.

If $w \in L(G)$ then the sequence
$S \Rightarrow w_1 \Rightarrow w_2 \Rightarrow \cdots \Rightarrow w_n \Rightarrow w$
is a derivation of string w. The strings $S, w_1, w_2, \ldots, w_n$ which contain variables as well as terminals are called sentential forms of derivation.

Example.

Consider the grammar $G = (\{S\}, \{a, b\}, S, P)$, with P: $S \rightarrow aSb$, $S \rightarrow \lambda$
then: $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$
so we can write: $S \overset{*}{\Rightarrow} aabb$.
The string aabb is a string generated by G while aaSbb is a sentential form.

OKESOLA AJIBOLA

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