Udofia, Etimbuk Victory 19/MHS01/410 MBBS

1.

1. $\int \sin^6 x$

$= \int (\sin^2 x)^3$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\sin^6 x = \left[\frac{1 - \cos 2x}{2}\right]^3$

$\int \sin^6 x = \frac{1}{8} \int (1 - \cos 2x)^3$

$\int \sin^6 x = \frac{1}{8} \int (1 - 2\cos 2x + \cos^2 2x)(1 - \cos 2x)$

$= \frac{1}{8} \int (1 - \cos 2x - 2\cos 2x + 2\cos^2 2x + \cos^2 2x - \cos^3 2x)$

$= \frac{1}{8} \int 1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x \, dx$

$= \frac{1}{8} \int 1 - 3\cos 2x + 3\left(\frac{1 + \cos 4x}{2}\right) - \cos 2x (1 - \sin^2 2x)$

$= \frac{1}{8} \int \frac{5}{2} - 4\cos 2x + \frac{3\cos 4x}{2} + \cos 2x \sin^2 2x \, dx$

$= \frac{1}{8} \left[\frac{5x}{2} - 2\sin 2x + \frac{3\sin 4x}{8} + \frac{\sin^3 2x}{6}\right] + C$

$= \frac{x}{2} - \frac{\sin 2x}{4} + \frac{3\sin 4x}{64} + \frac{\sin^3 2x}{48} + C$

2.

2. $\int \cos^4 x \sin^3 x$

$u = \cos x$

$du/dx = -\sin x$

$du = -\sin x \, dx, \quad dx = \frac{du}{-\sin x}$

$\sin^2 x = 1 - \cos^2 x$

$\int \cancel{\sin x} \sin^2 x \times u^4 - \frac{du}{\cancel{\sin x}}$

$\int \cos^4 x \sin^3 x = -\int (1 - \cos^2 x) u^4 \, du$

$\int \cos^4 x \sin^3 x = -\int (1 - u^2) u^4 \, du$

$\int \cos^4 x \sin^3 x = -\int (u^4 - u^8) du$

$\int \cos^4 x \sin^3 x = -\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + C$

$\int \cos^4 x \sin^3 x = \frac{u^7}{7} - \frac{u^5}{5} + C = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$

3.

3. $\int \cos x \sin^3 x \, dx$ where both are odd

$u = \sin x = \frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$= \cancel{\int u^3} \times \quad = \int \cancel{\cos x} \times u^3 \times \frac{du}{\cancel{\cos x}}$

$= \int u^3 \, du$

$\int \cos x \sin^3 x = \frac{u^4}{4} + c$

$\int \cos x \sin^3 x = \frac{\sin^4 x}{4} + c$