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MAT 104

Differentiation:

1a) $y = \dfrac{(x+1)^2(x-2)^{\frac{1}{2}}}{(2x-1)(x-3)^{\frac{3}{2}}}$

$\ln y = \left[\ln(x+1)^2 + \ln(x-2)^{\frac{1}{2}}\right] - \left[\ln(2x-1) + \ln(x-3)^{\frac{3}{2}}\right]$

$\dfrac{1}{y}\cdot\dfrac{dy}{dx} = \left[\dfrac{1}{(x+1)^2}\cdot 2(x+1) + \dfrac{1}{(x-2)^{\frac{1}{2}}}\cdot\dfrac{(x-2)^{-\frac{1}{2}}}{2}\right] - \left[\dfrac{1}{(2x-1)}\cdot 2 + \dfrac{1}{(x-3)^{\frac{3}{2}}}\cdot\dfrac{3(x-3)^{\frac{1}{2}}}{2}\right]$

$\dfrac{1}{y}\cdot\dfrac{dy}{dx} = \left[\dfrac{2(x+1)}{(x+1)^2} + \dfrac{(x-2)^{-\frac{1}{2}}}{2(x-2)^{\frac{1}{2}}}\right] - \left[\dfrac{2}{(2x-1)} + \dfrac{3(x-3)^{\frac{1}{2}}}{2(x-3)^{\frac{3}{2}}}\right]$

$\dfrac{1}{y}\cdot\dfrac{dy}{dx} = \left[\dfrac{2}{(x+1)} + \dfrac{1}{2(x-2)}\right] - \left[\dfrac{2}{(2x-1)} + \dfrac{3}{2(x-3)^2}\right]$

$\dfrac{dy}{dx} = y\left[\dfrac{2}{(x+1)} + \dfrac{1}{2(x-2)}\right] - \left[\dfrac{2}{(2x-1)} + \dfrac{3}{2(x-3)^2}\right]$

$\dfrac{dy}{dx} = \left\{\dfrac{(x+1)^2(x-2)^{\frac{1}{2}}}{(2x-1)(x-3)^{\frac{3}{2}}}\right.\left[\dfrac{2}{(x+1)} + \dfrac{1}{2(x-2)} \dfrac{-2}{(2x-1)} - \dfrac{3}{2(x-3)^2}\right]$

b) $y = \dfrac{3e^x \sin 2x}{x^{\frac{5}{2}}}$

$\ln y = (\ln 3e^x + \ln \sin 2x) - \ln x^{\frac{5}{2}}$

$\dfrac{1}{y}\cdot\dfrac{dy}{dx} = \left(\dfrac{1}{3e^x}(3e^x) + \dfrac{1}{\sin 2x}(\cos 2x)\right) - \dfrac{1}{x^{\frac{5}{2}}}\left(\frac{5}{2}x^{\frac{3}{2}}\right)$

$\dfrac{1}{y}\cdot\dfrac{dy}{dx} = \left(\dfrac{3e^x}{3e^x} + \dfrac{\cos 2x}{\sin 2x}\right) - \dfrac{\frac{5}{2}x^{\frac{3}{2}}}{x^{\frac{5}{2}}}$

$\cancel{y}\cdot\dfrac{dy}{dx} = y\left[\left(1 + \dfrac{\cos 2x}{\sin 2x}\right) - \dfrac{\frac{5}{2}x^{\frac{3}{2}}}{x^{\frac{5}{2}}}\right]$

$\dfrac{dy}{dx} = \dfrac{3e^x \sin 2x}{x^{\frac{5}{2}}}\left[\left(1 + \dfrac{\cos 2x}{\sin 2x}\right) - \dfrac{\frac{5}{2}x^{\frac{3}{2}}}{x^{\frac{5}{2}}}\right]$

Integration 1

2a) $\int 4\sec^2(3m+1)\,dm$

$u = 3m+1$

$du = 3dm$

$dm = \frac{du}{3}$

$= \int 4\sec^2 u \cdot \frac{du}{3}$

$= \frac{4}{3}\int \sec^2 u\,du$

$= \frac{4}{3}\tan u + c$

$= \frac{4}{3}\tan(3m+1) + c$

b) $\int 2t \times (3t^2-1)^{\frac{1}{2}}$

$u = 3t^2 - 1$

$\frac{du}{dt} = 6t$

$\Rightarrow dt = \frac{du}{6t}$

$\int 2t \times u^{\frac{1}{2}} \cdot \frac{du}{6t}$

$\int \frac{1}{3} \times u^{\frac{1}{2}} \cdot du$

$\frac{1}{3}\int u^{\frac{1}{2}} \cdot du$

$= \frac{1}{3} \times \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$

$= \frac{1}{3} \times \frac{2}{3} u^{\frac{3}{2}} + c$

$= \frac{2}{9} u^{\frac{3}{2}} + c$

$= \frac{2}{9}(3t^2-1)^{\frac{3}{2}} + c$

c) $\int \frac{2x}{(4x^2-1)^{\frac{1}{2}}} = \int 2x(4x^2-1)^{-\frac{1}{2}} \cdot dx$

$u = 4x^2 - 1$

$du = 8x \cdot dx$

$dx = \frac{du}{8x}$

$= \int 2x(u)^{-\frac{1}{2}} \cdot \frac{du}{8x}$

$= \frac{1}{4}\int u^{-\frac{1}{2}} \cdot du$

$= \frac{1}{4} \times \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$

$= \frac{1}{4} \times \frac{u^{\frac{1}{2}}}{\frac{1}{2}}$

$= \frac{1}{4} \times 2u^{\frac{1}{2}} = \frac{1}{2}u^{\frac{1}{2}} = \frac{1}{2}(4x^2-1)^{\frac{1}{2}} + c$