Questions

$$A = \begin{pmatrix} 1 & 4 & 8 \\ -3 & 0 & 5 \\ 6 & 2 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -2 & 3 \\ 2 & 1 & -4 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 6 & 1 \\ 4 & -7 & -3 \\ 3 & 1 & 2 \end{pmatrix}$$

Find (i) Rank of A
(ii) Rank of B transpose
(iii) Rank of (A+C) transpose
(iv) Rank of (B+C)
(v) Rank of (A+B+C)

(i) Rank of A

$$A = \begin{pmatrix} 1 & 4 & 8 \\ -3 & 0 & 5 \\ 6 & 2 & 1 \end{pmatrix}$$

$$\text{Rank} \begin{bmatrix} 1 & 4 & 8 \\ -3 & 0 & 5 \\ 6 & 2 & 1 \end{bmatrix}$$

now reducing this matrix interchanging $R_1 \Longleftrightarrow R_3$

$$\begin{bmatrix} 6 & 2 & 1 \\ -3 & 0 & 5 \\ 1 & 4 & 8 \end{bmatrix}$$

$R_1 \Leftarrow R_1 \div 6$

$$= \begin{bmatrix} 1 & 0.333 & 0.167 \\ -3 & 0 & 5 \\ 1 & 4 & 8 \end{bmatrix}$$

$R_2 \Leftarrow R_2 + 3 \times R_1$

$$\begin{bmatrix} 1 & 0.333 & 0.167 \\ 0 & 1 & 5.5 \\ 1 & 4 & 8 \end{bmatrix}$$

$R_3 \Leftarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 0.333 & 0.167 \\ 0 & 1 & 5.5 \\ 0 & 3.67 & 7.83 \end{bmatrix}$$

Interchanging rows $R_2 \Leftarrow\Rightarrow R_3$

$$\begin{bmatrix} 1 & 0.333 & 0.167 \\ 0 & 3.67 & 7.83 \\ 0 & 1 & 5.5 \end{bmatrix}$$

$= R_2 \Leftarrow R_2 \times 0.272$

$$= \begin{bmatrix} 1 & 0.33 & 0.167 \\ 0 & 1 & 2.14 \\ 0 & 1 & 5.5 \end{bmatrix}$$

$R_3 \Leftarrow R_3 - R_2$

$$= \begin{bmatrix} 1 & 0.33 & 0.167 \\ 0 & 1 & 2.14 \\ 0 & 0 & 3.40 \end{bmatrix}$$

$R_3 \Leftarrow R_3 \times 0.2973$

$$\begin{bmatrix} 1 & 0.33 & 0.167 \\ 0 & 1 & 2.14 \\ 0 & 0 & 1 \end{bmatrix}$$

The rank of a matrix is the number of non-all zeros rows

Rank = 3

ii) Rank Of B transpose

$$B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -2 & 3 \\ 2 & 1 & -4 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 3 & 4 \end{pmatrix}^T$$

$$= \begin{bmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 3 & 4 \end{bmatrix}^T$$ Rank

now, reduce this matrix

$R_2 \leftarrow R_2 - R_1$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 3 & 4 \end{bmatrix}$$

$R_2 \leftarrow R_2 \div -3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -0.33 \\ 0 & 3 & 4 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 3 \times R_2$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -0.33 \\ 0 & 0 & 5 \end{bmatrix}$$

$R_3 \leftarrow R_3 \div 5$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -0.33 \\ 0 & 0 & 1 \end{bmatrix}$$

The rank of a matrix Is the number of non all-zero rows

$\therefore$ Rank $= 3$

$$A+C = \begin{bmatrix} 1 & 10 & 9 \\ 1 & -7 & 2 \\ 3 & 1 & -1 \end{bmatrix}$$

$$\cancel{(A+C)^T = \begin{bmatrix} 1 & 1 & 3 \end{bmatrix}}$$

$$(A+C)^T = \begin{bmatrix} 1 & 1 & 3 \\ 10 & -7 & 1 \\ 9 & 2 & -1 \end{bmatrix}$$

Rank of matrix

$$(A+C)^T = \begin{pmatrix} 1 & 1 & 3 \\ 10 & -7 & 1 \\ 9 & 2 & 1 \end{pmatrix}$$

now reduce this matrix interchanging rows $R_1 \longleftrightarrow R_2$

$$\begin{bmatrix} 10 & -7 & 1 \\ 1 & 1 & 3 \\ 9 & 2 & 1 \end{bmatrix}$$

$R_1 \leftarrow R_1 \div 10$

$$\begin{bmatrix} 1 & -0.7 & 0.1 \\ 1 & 1 & 3 \\ 9 & 2 & 1 \end{bmatrix}$$

$R_2 \leftarrow R_2 - R_1$

$$= \begin{bmatrix} 1 & -0.7 & 0.1 \\ 0 & 1.7 & 2.9 \\ 9 & 2 & 1 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 9 \times R_1$

$$= \begin{bmatrix} 1 & -0.7 & 0.1 \\ 0 & 1.7 & 2.9 \\ 0 & 8.3 & 0.1 \end{bmatrix}$$

Interchanging rows $R_2 \Leftrightarrow R_3$

$$= \begin{bmatrix} 1 & -0.7 & 0.1 \\ 0 & 8.3 & 0.1 \\ 0 & 1.7 & 2.9 \end{bmatrix}$$

$R_2 \leftarrow R_2 \times 0.1205$

$$= \begin{bmatrix} 1 & -0.7 & 0.1 \\ 0 & 1 & 0.012 \\ 0 & 1.7 & 2.9 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 1.7 \times R_2$

$$= \begin{bmatrix} 1 & -0.7 & 0.1 \\ 0 & 1 & 0.012 \\ 0 & 0 & 2.88 \end{bmatrix}$$

$R_3 \leftarrow R_3 \times 0.3473$

$$= \begin{bmatrix} 1 & -0.7 & 0.1 \\ 0 & 1 & 0.012 \\ 0 & 0 & 1 \end{bmatrix}$$

The rank of a matrix is the number of non all-zeros rows

Rank = 3

(iv) Rank of (B+C)

(B+C)

$$B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -2 & 3 \\ 2 & 1 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 6 & 1 \\ 4 & -7 & -3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$(B+C) = \begin{pmatrix} 1 & 7 & 1 \\ 5 & -9 & 0 \\ -1 & 0 & 2 \end{pmatrix}$$

Rank of matrix of (B+C)
now reducing this matrix interchanging rows $R_1 \Leftrightarrow R_2$

$$= \begin{bmatrix} 5 & -9 & 0 \\ 1 & 7 & 1 \\ -1 & 0 & 2 \end{bmatrix}$$

$R_1 \leftarrow R_1 \div 5$

$$= \begin{bmatrix} 1 & -1.8 & 0 \\ 1 & 7 & 1 \\ -1 & 0 & 2 \end{bmatrix}$$

$R_3 \leftarrow R_3 \cancel{R_1} - R_1$

$$\begin{bmatrix} 1 & -1.8 & 0 \\ 0 & 8.8 & 1 \\ -1 & 0 & 2 \end{bmatrix}$$

$R_3 \leftarrow R_3 + R_1$

$$= \begin{bmatrix} 1 & -1.8 & 0 \\ 0 & 8.8 & 1 \\ 0 & -1.8 & 2 \end{bmatrix}$$

$R_2 \leftarrow R_2 \times 0.1136$

$$= \begin{bmatrix} 1 & -1.8 & 0 \\ 0 & 1 & 0.1136 \\ 0 & -1.8 & 2 \end{bmatrix}$$

$R_3 \leftarrow R_3 + 1.8 \times R_2$

$$= \begin{bmatrix} 1 & -1.8 & 0 \\ 0 & 1 & 0.1136 \\ 0 & 0 & 2.205 \end{bmatrix}$$

$R_3 \leftarrow R_3 \times 0.4536$

$$= \begin{bmatrix} 1 & -1.8 & 0 \\ 0 & 1 & 0.1136 \\ 0 & 0 & 1 \end{bmatrix}$$

The rank of a matrix is the number of nonall zeros row

$\therefore$ Rank $= 3$

(V) Rank of A+B+C

$$A = \begin{pmatrix} 1 & 4 & 8 \\ -3 & 0 & 5 \\ 6 & 2 & 1 \end{pmatrix} + B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -2 & 3 \\ 2 & 1 & 4 \end{pmatrix}$$

$$+ C = \begin{pmatrix} 0 & 6 & 1 \\ 4 & -7 & -3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$(A+B+C) = \begin{pmatrix} 2 & 11 & 9 \\ 2 & -9 & 5 \\ 11 & 4 & 7 \end{pmatrix}$$

$$\text{Rank} = \begin{bmatrix} 2 & 11 & 9 \\ 2 & -9 & 5 \\ 11 & 4 & 7 \end{bmatrix}$$

now reduce this Matrix

$$= \begin{bmatrix} 11 & 4 & 7 \\ 2 & -9 & 5 \\ 2 & 11 & 9 \end{bmatrix}$$

$R_1 \leftarrow R_1 \div 11$

$$= \begin{bmatrix} 1 & 0.364 & 0.636 \\ 2 & -9 & 5 \\ 2 & 11 & 9 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2 \times R_1$

$$\begin{bmatrix} 1 & 0.3636 & 0.6364 \\ 0 & -9.7273 & 3.7273 \\ 2 & 11 & 9 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 2 \times R_1$

$$= \begin{bmatrix} 1 & 0.3636 & 0.6364 \\ 0 & -9.7273 & 3.7273 \\ 0 & 10.2727 & 7.7273 \end{bmatrix}$$

Interchanging row $R_2 \longleftrightarrow R_3$

$$\begin{bmatrix} 1 & 0.3636 & 0.6364 \\ 0 & 10.2727 & 7.7273 \\ 0 & -9.7273 & 3.7273 \end{bmatrix}$$

$R_2 \leftarrow R_2 \times 0.0973$

$$= \begin{bmatrix} 1 & 0.3636 & 0.6364 \\ 0 & 1 & 0.7522 \\ 0 & -9.7273 & 3.7273 \end{bmatrix}$$

$R_3 \leftarrow R_3 + 9.7273 \times R_2$

$$\begin{bmatrix} 1 & 0.3636 & 0.6364 \\ 0 & 1 & 0.7522 \\ 0 & 0 & 11.0442 \end{bmatrix}$$

$R_3 \leftarrow R_3 \div 11.0442$

$$\begin{bmatrix} 1 & 0.3636 & 0.6364 \\ 0 & 1 & 0.7522 \\ 0 & 0 & 1 \end{bmatrix}$$

The rank of a matrix is the number of non all-zeros rows

$\therefore$ Rank = 3

$$\begin{bmatrix} 6 & 2 & 1 \\ -3 & 0 & 5 \\ 1 & 4 & 8 \end{bmatrix}$$

$R_1 \Leftarrow R_1 \div 6$

$$= \begin{bmatrix} 1 & 0.333 & 0.167 \\ -3 & 0 & 5 \\ 1 & 4 & 8 \end{bmatrix}$$

$R_2 \Leftarrow R_2 + 3 \times R_1$

$$\begin{bmatrix} 1 & 0.333 & 0.167 \\ 0 & 1 & 5.5 \\ 1 & 4 & 8 \end{bmatrix}$$

$R_3 \Leftarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 0.333 & 0.167 \\ 0 & 1 & 5.5 \\ 0 & 3.67 & 7.83 \end{bmatrix}$$

Interchanging rows $R_2 \Longleftrightarrow R_3$

$$\begin{bmatrix} 1 & 0.333 & 0.167 \\ 0 & 3.67 & 7.83 \\ 0 & 1 & 5.5 \end{bmatrix}$$

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$$= \begin{bmatrix} 1 & 0.33 & 0.167 \\ 0 & 1 & 2.14 \\ 0 & 1 & 5.5 \end{bmatrix}$$

$R_3 \Leftarrow R_3 - R_2$

$$= \begin{bmatrix} 1 & 0.33 & 0.167 \\ 0 & 1 & 2.14 \\ 0 & 0 & 3.40 \end{bmatrix}$$

$R_3 \Leftarrow R_3 \times 0.2973$

$$\begin{bmatrix} 1 & 0.33 & 0.167 \\ 0 & 1 & 2.14 \\ 0 & 0 & 1 \end{bmatrix}$$

The rank of a matrix is the number of non-all zeros rows

Rank = 3