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18/SCI01/099

1) Computability theory is the part of Theory of computation dealing with which problems are solvable by algorithms (generally by Turing machines) with various restrictions and extensions.

~~Computability~~ while Complexity theory is a part of the Theory of computation dealing with the resources required during computation to solve a given problem. The most important resources common resources are time, how many parallel processors are needed to solve a problem in parallel etc.

Complexity theory differs from Computability theory, which deals with whether a problem can be solved at all, regardless of the resources required.

2) Computability theory deals with the question of whether a problem is solvable at all on a computer. Computability theory addresses

A main question (i) What problems can a Turing machine solve

(ii) What other systems are equivalent to Turing machines

(iii) What problems require more powerful machines

(iv) What problems can be solved with less powerful machines.

E.g. an addition problem of $2+2$ is solvable

ii) Complexity theory amongst other things investigates the solvability of computational problems and algorithms. In particular, it

places practical limits on what computers can or cannot accomplish.

E.g. Mowing grass has linear complexity because it takes double the time to mow double the area. However, looking up something

in a dictionary has only logarithmic complexity because for a

double sized dictionary you have to open it one time more (i.e. exactly in the middle then the problem is reduced to half.)

3) In mathematics, a set is a well defined collection of distinct objects, considered as an object in its own right.

e.g. the set A of all natural numbers from 1 to 4 $A = \{1, 2, 3, 4\}$

ii) In mathematics, the power set of any set S is the set of all subsets of S including the empty set and S itself.

e.g. if S is the set $\{x, y, z\}$

then the subsets of S are

$\{\emptyset\}$

$\{x\}$

$\{y\}$

$\{z\}$

$\{x, y\}$

$\{x, z\}$

$\{y, z\}$

$\{x, y, z\}$ and hence the power^{set} of S is

$\{\emptyset, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}$

iii) In mathematics, an element or member of a set is any one of the distinct objects that make up the set e.g. A set B contains $\{1, 2, 3, 4, 5\}$, 2 is an element or member of the set B .

iv) In mathematics a set A is a subset of a set B , or equivalently B is a superset of A if A is contained in B .

e.g. $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5, 6\}$ $\therefore A$ is a subset of B

v) In mathematics a proper subset of a set A is a subset of A that is not equal to A i.e. if B is a proper subset of A , then all elements of B are in A but A consists of at least one element that is not in B e.g. $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{1, 2, 3\}$

$\therefore B$ is a proper subset of A .

vi) In mathematics, an infinite set is a set which has uncountable elements. Therefore it is represented by an ellipse at the end of the set to show continuity e.g. The set of all natural numbers: $A = \{1, 2, 3, 4, \dots\}$

vii) In mathematics, a finite set is a set that has a finite set of elements e.g. A is a set of all natural numbers from 1 to 5
 $A = \{1, 2, 3, 4, 5\}$

viii) In mathematics, an unordered pair or pair set is a set of the form $\{a, b\}$, i.e. a set having two elements a and b with no particular relation between them e.g. $a \neq b$

ix) In set theory, the union (denoted by \cup) of a collection of sets is the set of all elements in the collection.
eg set $A = \{1, 2, 3, 5\}$ set $B = \{4, 6, 7, 8\}$
 $\therefore A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

x) In mathematics, the intersection of two sets A and B denoted by $A \cap B$, is the set containing all elements of A that also belong to B e.g. set $A = \{1, 2, 3, 5\}$ set $B = \{1, 4, 5, 6, 7\}$
 $\therefore A \cap B = \{1, 5\}$

xi) In set theory, the complement of a set A , denoted by A' is the set of all elements in the given universal set U that are not in A e.g. $U = \{1, 2, 3, 4, 5, \dots\}$ and $A = \{1, 2\}$
 $A' = \{3, 4, 5, \dots\}$

xii) In set theory the difference of two sets A and B denoted by $(A - B)$ is the set of all elements that are in A but not in B . e.g. $A = \{1, 2, 3, 6\}$
 $B = \{1, 3, 5, 7\}$ $\therefore A - B = \{2, 6\}$

xiii) In mathematics, the symmetric difference also known as the disjunctive union of two sets is the set of elements which are ⁱⁿ either of the sets and not in their intersection.

eg Set A = {1, 2, 3, 4, 5} and set B = {1, 3, 5, 7, 9}

$$A \oplus B = (A - B) \cup (B - A)$$

$$= (2, 4) \cup (7, 9)$$

$$A \oplus B = (2, 4, 7, 9)$$

4) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$$A = \{2, 3, 5, 6, 9\}$$

$$B = \{0, 2, 4, 8, \dots, 20\}$$

$$C = \{1, 3, 5, \dots\}$$

$$D = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

i) $B \cap C = \{9, 10, 11, 12, \dots, 20\}$

ii) $A \cap D = \{2, 3, 5, 6, 9\}$

$$A \cap C = \{3, 5, 6, 9\}$$

iii) $B^c = \{1, 3, 5, 6, 7\}$

$$D^c = \{E\}$$

$$C^c = \{2, 4, \dots\}$$

iv) $A \oplus C = \{1, 2, 7, 8, 10, \dots\}$

$$B \oplus C = \{0, 1, 2, 3, 4, 5, 6, 7, \dots, 21, \dots\}$$

$$D \oplus B = \{0, 10, 11, \dots, 20\}$$

v) $A \cup B = \{0, 2, 3, 4, 5, 6, 8, 9, \dots, 20\}$

$$B \cup D = \{0, 1, 2, 3, \dots, 20\}$$

$$C \cup D = \{1, 2, 3, \dots\}$$

$$\begin{aligned} \vee \quad A - B &= \{3, 5, 6\} \\ D - C &= \{2, 4\} \\ C - B &= \{1, 3, 5, 6, 7, 2, \dots\} \end{aligned}$$

5) i) Alphabets

Theory of computation is entirely based on symbols. These symbols are generally letters and digits. Alphabets are defined as a finite set of symbols. E.g.

$\Sigma = \{0, 1\}$ is an alphabet of binary digits.

ii) Words

Consider a non-empty set Σ of symbols. A word on the set Σ is a finite sequence of its elements.

iii) Length of a word

The length of a word U written as $|U|$ or (U) is the no. of elements in its sequence of letters.

iv) Substring

Consider any word ' U ' on an alphabet Σ . Any sequence $W = a_i a_{i+1} \dots a_k$ is called a substring of U .

v) Initial segment

The substring $W = a_1 a_2 \dots a_k$ beginning with the first letter of ' U ' is called an initial segment of U .

vi) Concatenation of strings

Concatenating two strings U & V on the alphabet Σ , the concatenation written as UV is the word obtained by writing down the letters of U followed by the letters of V .

iii) language

A language "L" over an alphabet " Σ " is a collection of strings/words on Σ .

6) $A = \{a, c, e, i, o, n, t\}$

\vee = concatenation

u = concatenation

i) length of $\vee = 13$

ii) $\vee u \vee = \text{concatenationcatetioncong}$

iii) $u^R = \text{anocnoiteta c}$

iv) $m = \text{c a c o n a t i o n e}$

$|m| = 12$

Substring of $m = \Sigma^0 u \Sigma^1 u \Sigma^2 u \Sigma^3 \dots u \Sigma^{12}$

$\Sigma^0 = \{\lambda\}$

$\Sigma^1 = \{c, a, o, t, i, e, n\}$

$\Sigma^2 = \{ca, to, ie, nt\}$

$\Sigma^3 = \{cao, tie, tin\}$

$\Sigma^4 = \{caot, tien\}$

$\Sigma^5 = \{caoti, otien\}$

$\Sigma^6 = \{caotie, aotien\}$

$\Sigma^7 = \{caotien, netoac\}$

$\Sigma^8 = \{caotien c, aotienca\}$

$\Sigma^9 = \{caotienca, catotienca\}$

$\Sigma^{10} = \{canietoaca, catcaotienca\}$

$\Sigma^{11} = \{canotienaci, cacinotienca\}$

$\Sigma^{12} = \{cicanotienai, canacinotienca\}$

$m = \{\lambda, c, ca, cao, caot, caoti, caotie, caotien, caotien c, caotienca, canietoaca, canotienaci, cicanotienai\}$

7 $S = abracaba$

i) $|S| = 8$

$$\Sigma^0 = \{ \lambda \}$$

$$\Sigma^1 = \{ a, b, r, c \}$$

$$\Sigma^2 = \{ ab, rc \}$$

$$\Sigma^3 = \{ abr, brc \}$$

$$\Sigma^4 = \{ abrc, barc \}$$

$$\Sigma^5 = \{ abrcr, barcr \}$$

$$\Sigma^6 = \{ abrcr, abrcar \}$$

$$\Sigma^7 = \{ bracaba, bacarbr \}$$

$$\Sigma^8 = \{ abracaba, abrcarba \}$$

$$S = \{ \lambda, a, ab, abr, abrc, abrcr, abrcr, bracaba, abracaba \}$$

ii) ~~The~~ The initial segment is a

8) i)

START - t

END - z

Set of all english words that start with the letter t

Set of all english words that end with z

ii)

A ∩ B

Set of all english words that start with the letter t

Set of all words that start with t and also end with z

Set of all english words that end with z

iii)

A ∪ B

Set of all the words that start with t and end with z

i) Set of strings containing 11-n it
= {011, 11, 111, 0111, 1111, ...}

ii) Set of all strings starting with 0 and ending with 1
= {01, 011, 001, 0101, 0011, ...}

iii) Set of all strings with length = 2
= {01, 10, 00, 11}

iv) Set of all strings ending with 10
= {10, 0110, 0010, 01010, 110, ...}

i) A = {e, n, o, u, g, h}

ii) A = {b, e, a, r, 2, o}

iii) A = {0, 1}

i) A = {0, p}

i) Set of all strings ending with p
= {op, oop, pop, p o o p, p, ...}

ii) Set of all strings with equal number of 0's and p's
= {op, oopp, ooo p p p, o p o p o p, ...}

iii) Set of all strings starting with p and ending with 0
= {p0, p o p o, p p o o, p o p p o, ...}

iv) set of all strings starting with p ^{length 4} and ending with q
= $\{0000, pppp, opop, poop, ooop, \dots\}$

12) i) $\{b^n a \mid n > 0\}$

set of all strings beginning with one or more b's followed by an a

ii) $\{a^n a b^m \mid n > 0, m > 0\}$

set of all strings beginning with zero or more a's followed by one ~~or~~ a followed by one or more b's.

iii) $\{b^n b^m a^m \mid n > 0, m > 0\}$

set of all strings beginning with zero or more b's followed by zero or more b's followed by zero or more a's

iv) $\{ab, abb, eab, aqa, bbbab, \dots\}$
 $\{b^p a^m b^n \mid p > 0, m > 0, n > 0\}$

set of all strings beginning with zero or more b's followed by one or more a's followed by zero or more b's

13) $A = \{a, b\}$

i) $\{b, ba, bab, baa, bbb, \dots\}$

$\{b^n a^m b^p \mid n > 0, m > 0, p > 0\}$

set of all strings beginning with ~~one~~ ^{one} or more b's followed by zero or more a's followed by zero or more b's

ii) $\{bas, babb, bbab, bba'bb, ba'bbb, bbbab, \dots\}$
 $b^m a b^n \mid m > 0, n > 0$

set of all strings starting with one or more b's followed by one a followed by one or more b's

iii) $\{b, bb, bbb, bbbb, \dots\}$

$\{b^n \mid n > 0\}$

Set of all strings starting with one or more b's

iv) $\{ab, aabb, aaaa bbb, \dots\}$

$\{a^n b^n \mid n > 0\}$

Set of all strings starting with one or more a's,
followed by one or more b's

1+) Union - $L_1 \cup L_2 = \{u \in \Sigma^* \mid u \in L_1, \text{ or } u \in L_2\}$

Intersection - $L_1 \cap L_2 = \{u \in \Sigma^* \mid u \in L_1, \text{ and } u \in L_2\}$

Difference - $L_1 - L_2 = \{u \in \Sigma^* \mid u \in L_1, \text{ and } u \notin L_2\}$

Complement - $\overline{L} \text{ or } L^c = \{\Sigma^* - L\}$

Positive Closure - $L^+ = \bigcup_{i=1}^{\infty} L^i = \{L^1 \cup L^2 \cup \dots\}$

Star Operation - $L^* = \bigcup_{i=0}^{\infty} L^i = \{L^0 \cup L^1 \cup L^2 \cup \dots \cup L^i\}$

Multiplication - $L_1 L_2 = \{uv \mid u \in L_1, v \in L_2\}$

Power - $L^0 = \{\epsilon\}$, $L^n = L^{n-1} L$ if $n \geq 1$