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15) i) Length is at most 2

$$L = \{ \epsilon, a, b, ab, aa, ba, bb \}$$
$$(\epsilon + a + b)(\epsilon + a + b)$$

ii) Language of even length

$$L = \{ aa, bb, aabb, aaabbb, \dots \}$$
$$((a+b)(a+b))^*$$

iii) Language starting and ending with the same letter

$$a(a+b)^*a$$

iv) Language starting and ending with different letters

$$a(a+b)^*b + b(a+b)^*a$$

16) Let  $L$  be a language over an alphabet, then  $L$  is a regular language if and only if there exists a regular expression  $R$  over an alphabet such that  $L = L(R)$ . A regular expression is a sequence of characters that define a search pattern.

Identities of  $R, \epsilon$

i)  $\emptyset + R = R$

ii)  $\emptyset R + R\emptyset = \emptyset$

iii)  $\epsilon R = R\epsilon = R$

iv)  $\epsilon^* = \epsilon$  and  $\emptyset^* = \epsilon$

v)  $R + R = R$

vi)  $R^* R^* = R^*$

vii)  $RR^* = R^*R$

$$viii) (R^*)^* = R^*$$

$$ix) \epsilon + RR^* = \epsilon + R^*R = R^*$$

$$x) (PQ)^*P = P(QP)^*$$

$$xi) (P+Q)^* = (P^*Q^*)^* = (P^*+Q^*)^*$$

$$xii) (P+Q)R = PR+QR \text{ and } R(P+Q) = RP+RQ$$

17) Let  $A$  be a non empty alphabet the expression  $r$  and its corresponding language  $L(r)$  are defined inductively as follows. The symbol  $\lambda$  and pair  $()$  are regular expressions, each letter  $a$  in  $A$  is a regular expression. If  $r$  is a regular expression  $r^*$  is a regular expression if  $r_1$  and  $r_2$  are regular then  $r_1 \cup r_2$  is a R.E. If  $r_1$  and  $r_2$  are R.E then  $r_1, r_2$  are R.E.

ii) The symbols used in regular expression are  $\{\epsilon, *, \cup, \lambda, ()\}$

$$18) i) r = b^*$$

The language  $L(r)$  consists of all  $b$ 's including  $\epsilon$   
e.g.  $\{\epsilon, b, bb, bbb, \dots\}$

$$ii) \text{ Let } r = aa^*$$

The language  $L(r)$  consists of all positive powers of  $a$  excluding the empty word.

$$iii) \text{ Let } r = aub^*$$

The language  $L(r)$  consists of  $a$  or any word in  $b$ .

$$iv) \text{ Let } r = (aub)^*$$

The language  $L(r)$  consists of all words over the given alphabet  $A$

v) Let  $r = aa(aub)^*$

The language  $L(r)$  consists of all words starting with  $aa$

vi) Let  $r = (a+nb)^*$

The language  $L(r)$  consist of all combinations of  $a$  &  $b$

19)  $\Sigma = \{a, b\}$

$$L_1 = \{b^m a^n \mid m > 0, n > 1\}$$

$$L_2 = \{a^n b^m a \mid m > 1, n > 0\}$$

$$L_3 = \{a b^n \mid n > 0\}$$

Find a R.E.  $r$  over  $A = \{a, b\} \forall L$

$$L_i = L(r) \forall i = 1, 2, 3$$

$$L_1 = bb^* a a a^*$$

$$L_2 = a a^* b b b^* a$$

$$L_3 = a b b^*$$

20) A regular set is any set represented by regular expression eg  $a, b, \epsilon \in \Sigma$  then

$a$  denotes the set  $\{a\}$

$a+ b$  denotes the set  $\{a, b\}$

$ab$  denotes  $\{ab\}$

$a^*$  denotes the set  $\{\epsilon, a, aa, aaa, \dots\}$

$(a+b)^*$  denotes  $\{a, b\}^*$

The set represented by  $R$  is denoted by  $L(R)$ . Eg  
let  $R_1$  and  $R_2$  denote any two regular expression  
Then

1 A string  $L(R_1R_2)$  is a string from  $R_1$  followed by a string from  $R_2$

2 A string in  $L(R_1+R_2)$  is a string from  $R_1$  or a string from  $R_2$

3 A string in  $L(R^*)$  is a string obtained by concatenating  $n$  elements for some  $n \geq 0$

21  $\{110\}$

$\{1\}$   $\{0\}$  are represented by  $1$  &  $0$  respectively therefore  $110$  is obtained by concatenating  $1, 1$  and  $0$

ii)  $\{baab\}$

$\{b\}$   $\{a\}$  are represented by  $b$  and  $a$  respectively therefore  $baab$  is obtained by concatenating  $b, a, a$  and  $b$

iii)  $\{01, 10\}$

as  $\{01, 10\}$  is the union of  $\{01\}$  and  $\{10\}$  then we have it represented as  $01 + 10$

iv)  $\{\epsilon, 10\}$

The set  $\{\epsilon, 10\}$  is also represented by  $\epsilon + 10$

v)  $\{abb, a, b, bb\}$

The set  $\{abb, a, b, bb\}$  is also represented by  $abb + a + b + bb$

vi)  $\{\epsilon, a, aa, aaa, \dots\}$

R.E for this set is  $\epsilon + a + a^2 + \dots = a^*$

vii)  $\{1, 11, 111, \dots\}$

The set represents  $\{1\}^*$ . R.E for this set is  $1(1)^*$

22) i)  $L_1$  is the set of all strings of a's and b's ending in a a

$(a + b)^* a a$

ii)  $L_2$  is the set of all strings of 0's and 1's beginning with 1 and ending with 0

$1(1+0)^* 0$

iii)  $L_3$  is the set of  $\{\epsilon, 11, 111, 1111, \dots\}$

epsilon  $1(1)^*$

23) Grammar are finite set of rules used to describe languages. "Grammar is a generator of language while Language is a set of strings generated by grammar"

24) The sentential derivation of a string is any string derivable from the start symbol ~~at~~ during language generation. e.g Consider the grammar  $G = (\{S\}, \{a, b\}, S, P)$  with Production rules  $S \rightarrow a S b, S \rightarrow \lambda$

$S \rightarrow a S b \rightarrow a \lambda b \rightarrow ab$

This is a sentential derivation.