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$S \longrightarrow aSa \mid bSb \mid a \mid b \mid \lambda$

<u>solution</u>

$S \rightarrow aSa \rightarrow a\lambda a \rightarrow aa$

$S \rightarrow aSa \rightarrow aaa$

$S \rightarrow aSa \rightarrow aba$

$S \rightarrow aSa \rightarrow bSb \rightarrow b\lambda b \rightarrow bb$

$S \rightarrow aSa \rightarrow bSb \rightarrow bbb$

$S \rightarrow aSa \rightarrow abSba \rightarrow ab\lambda ba \rightarrow abba$

$S \rightarrow aSa \rightarrow bSb \rightarrow bbSbb \rightarrow bb\lambda bb \rightarrow bbbb$

$S \rightarrow aSa \rightarrow bSb \rightarrow bab$

$S \rightarrow aSa \rightarrow abSb \rightarrow ab\lambda b \rightarrow abb$

$S \rightarrow aSb \rightarrow a\lambda b \rightarrow ab$

$\{aa, aaa, aba, bb, bbb, abba, bbbb, bab, abb, ab, \ldots\}$

$\{a^m b^n \mid m \geq 0, n \geq 0\}$

26

$G = (V, T, S, P)$

i $S \rightarrow aS \mid bS \mid a \mid B$

<u>solution</u>

$S \rightarrow aS \rightarrow aa$

$S \rightarrow aS \rightarrow aB$

$S \rightarrow aS \rightarrow abS \rightarrow aba$

$S \rightarrow aS \rightarrow abS \rightarrow abB$

$\{aa, aB, aba, abB, \ldots\}$

ii $S \rightarrow aSa \mid bSb \mid aSb \mid \lambda$

Solution

$S \rightarrow aSa \rightarrow a\lambda a \rightarrow aa$

$S \rightarrow aSa \rightarrow abSba \rightarrow ab\lambda ba \rightarrow abba$

$S \rightarrow aSa \rightarrow aaSba \rightarrow aa\lambda ba \rightarrow aaba$

$S \rightarrow aSa \rightarrow abSba \rightarrow abaSbba \rightarrow aba\lambda bba$

$\rightarrow ababba$

$\{\lambda aa, abba, aaba, abababba, \ldots\}$

iii $S \rightarrow aAb \mid aBb \mid aSb, A \rightarrow aA \mid a, B \rightarrow bB \mid b$

Solution

$S \rightarrow aAb \rightarrow aab$

$S \rightarrow aAb \rightarrow aaAb \rightarrow aaab$

$S \rightarrow aBb \rightarrow abb$

$S \rightarrow aBb \rightarrow abBb \rightarrow abbb$

$S \rightarrow aSb \rightarrow aaAbb \rightarrow aaabb$

$\{aab, aaab, abb, abbb, aaabb, \ldots\}$

[7]

i $S \to aAb$, $A \to aA \mid bA \mid \lambda$

Solution

$S \to aAb \to a\lambda b \to ab$

$S \to aAb \to aaAb \to aa\lambda b \to aab$

$S \to aAb \to abAb \to ab\lambda b \to abb$

$S \to aAb \to aaAb \to abAb \to aab\lambda b \to$

$\cancel{S \to aAb} \to aabb.$

$\{\lambda, ab, aab, abb, aabb, \ldots\}$

$\{a^m b^n \mid m \geq 0, n \geq 0\}$

ii $S \to aSb \to aabb$

Solution

$S \to aSb \to aabb$

$S \to aSb \to aaSbb \to aaabbb$

$\{a^m b^n \mid m>0, n>0\}$, $\{aabb, aaabbb\}$

iii $S \to aSc \mid aAc$, $A \to aAb \mid ab$

Solution

$S \to aSc \to aaAcc \to aaabcc$

$S \to aSc \to aaAcc \to aaaAbcc$

$\to aaabbcc$

$S \to aSc \to aaAcc \to aaaAbcc \to aaaabbcc$

$\{a^{m+1}b^n c^0 \mid m>0, n>0, 0\}$ {aaabcc, aaaabbcc, aaaabbbcc, ...}

iv $S \to AB$, $B \to bB \mid b$, $A \to aA \mid a$

<u>Solution</u>

$S \to AB \to aB \to ab$

$S \to AB \to aAB \to aab$

$S \to AB \to aB \to abB \to abb$

$S \to AB \to aAB \to aaAB \to aaab$

$S \to AB \to aAB \to aaAB \to aaAbB \to$ aaabb

$S \to AB \to AbB \to abbB \to abbb$

{ab, aab, abb, aaab, aaabb, abbb, ...}

$\{a^{m+1} b^{n-1} \mid m>0, n>0\}$

Q8

$L(G) = \{a^n b^m \mid n \geq m\}$

L(G) consists of words starting with one or more 'a's followed by one or more 'b's but a will be greater or equal to b.

$S \rightarrow aS \mid bS \mid a \mid b$

i. babbaa

Solution

$S \rightarrow bS \rightarrow baS \rightarrow babS \rightarrow babbS \rightarrow babbaS$
$\rightarrow babbaa$

ii. babababa

Solution

$S \rightarrow bS \rightarrow baS \rightarrow babS \rightarrow babaS \rightarrow$
$bababS \rightarrow bababaS \rightarrow babababS \rightarrow$
$babababa.$

iii. aaabaa

Solution

$S \rightarrow aS \rightarrow aaS \rightarrow aaaS \rightarrow aaabS \rightarrow$
$aaabaS \rightarrow aaabaa.$

iv. baabaaa

Solution

$S \rightarrow bS \rightarrow baS \rightarrow baaS \rightarrow baabS \rightarrow$
$baabaS \rightarrow baabaaS \rightarrow baabaaa.$

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i. Mod 2=0

$(a+b)\ (a+b)^*$

ii Mod 2=1

$[(a+b)\ (a+b)]^*\ (a+b)$

iii Mod 3=0

$[(a+b)\ (a+b)\ (a+b)]^*$

iv Mod 3 = 2

$[(a+b)\ (a+b)\ (a+b)]^*\ (a+b)(a+b)$

v with the same power

$(a+b)^*$

vii Starting and ending with the same symbol.

$\$\ (a+b)^*\ \$$

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Automata theory is the study of abstract machines and automata, as well as the computational problems that can be solved using them.

It can be represented using DFA (Deterministic Finite Automaton) and NFA (Non-deterministic Finite Automaton).