Question 25

$S \to aSa \mid bSb \mid a \mid b \mid T$

Solution

$S \to aSa \to aTa \to aa$

$S \to aSa \to aaa$

$S \to aSa \to aba$

$S \to aSa \to bSb \to bTb \to bb$

$S \to aSa \to bSb \to bbb$

$S \to aSa \to abSb \to aSTba \to abba.$

$S \to aSa \to bSb \to bbSbb \to bbTbb$

$S \to aSa \to bSb \to bab$

$S \to aSa \to abSb \to abTb \to abb$

$S \to aSb \to aTb \to ab$

$\{aa, aaa, aba, bb, bbb, abba, bbbb, bab, ab, \ldots\}$

$\{a^m b^n \mid m \geq 0, n \geq 0\}$

Question 26

$G = (V, T, S, P)$

$S \to aS \mid bS \mid a \mid B$

Solution.

$S \to aS \to aa$

$S \to aS \to aB$

$S \to aS \to abS \to abc$

$S \to aS \to abS \to abB$

$\{aa, aB, aba, abB, \ldots\}$

ii $S \to aSa \mid bSb \mid asb \mid \lambda$

Solution:

$S \to aSa \to a\lambda a \to aa$

$S \to aSa \to abSba \to ab\lambda ba \to abba$

$S \to aSa \to aaSba \to aa\lambda ba \to aaba$

$S \to aSa \to abSba \to abaSbba \to aba\lambda bba$

$\to ababba.$

$\{\lambda, aa, abba, aaba, ababba, \ldots\}$

iii $S \to aAb \mid aBb \mid asb$, $A \to aAa$, $B \to bB \mid b$

Solution:

$S \to aAb \to aab$

$S \to aAb \to aaAb$

$a \to aBb \to abb$

$S \to aBb \to abBb \to abbb$

$S \to asb \to aaAb \to aaabb$

$\{aab, aaab, abb, abbb, aaabb, \ldots\}$

Question 27

i) $S \to aAb$, $a \to aA \mid bA \mid \lambda$

Solution

$S \to aAb \to a\lambda b \to ab$

$S \to aAb \to aaAb \to aa\lambda b \to aab$

$S \to aAb \to abAb \to ab\lambda b \to abb$

$S \to aAb \to aaAb \to aabAb \to aabAb \to aabb$

$\{\lambda, ab, aab, abb, aabb, \ldots\}$

$a^m b^n \mid m \geq 0, n \geq 0\}$.

ii) $S \to aSb \mid ab$

Solution

$S \to aSb \to aabb$

$S \to aSb \to aaSbb \to aaabbb$

$\{a^m b^n \mid m > 0, n > 0\}$, $\{aabb, aaabbb\}$

iii) $S \to aSc \mid aAc$, $A \to aAb \mid ab$

Solution

$S \to aSc \to aaAcc \to aaabcc$

$S \to aSc \to aaAcc \to aaaAbcc \to aaaabcc$

$S \to aSc \to aaAcc \to aaaAbcc \to aaaabbcc$

$\{a^{m+1} b^n c^0 \mid m \geq 0, n > 0\}$

$\{aaabcc, aaabbcc, aaaabbcc, \ldots\}$

iv) $S \to AB$, $B \to bB \mid b$, $A \to aA \mid a$

Solution.

$S \to AB \to ab \to ab$

$S \to AB \to aAB \to aab$

$S \to AB \to aB \to abB \to abb$

$S \to AB \to aAB \to aaAB \to aaab$

$S \to AB \to aAB \to aaAB \to aaAbB \to aaabb$

$S \to AB \to AbB \to abbB \to abbb$

$\{ab, aab, abb, aaab, aaabb, abbb, \ldots\}$

$\{a^{m+1} b^n \mid m > 0, n > 0\}$

Question 29

$S \rightarrow aS \mid bS \mid aS \mid b$

i) babbaa

Solution

$S \rightarrow bS \rightarrow baS \rightarrow babS \rightarrow babbS \rightarrow babbaS \rightarrow$ babbaa

ii) babababc

Solution

$S \rightarrow bS \rightarrow baS \rightarrow babS \rightarrow babaS \rightarrow bababS \rightarrow$ $bababaS \rightarrow babababS \rightarrow$ babababc.

iii) aaabaa

Solution

$S \rightarrow aS \rightarrow aaS \rightarrow aaaS \rightarrow aaabS \rightarrow aaabaS$ $\rightarrow$ aaabaa

iv) baabaa

Solution

$S \rightarrow bS \rightarrow baS \rightarrow baaS \rightarrow baabS \rightarrow baab$ $\rightarrow baabaaS \rightarrow baabaaa$ //

Question 30

i mod 2 = 0

$((a+b)(a+b))^*$

i mod 2 = 1

$((a+b)(a+b))^* (a+b)$

mod 3 = 0

$((a+b)(a+b)(a+b)^*)$

mod 3 = 2

$(a+b)(a+b)(a+b)^* (a+b)(a+b)$

) with the same power : $(a+b)^*$

iii) Starting and ending with the same symbol.

$S \mid (a+b)^* S$