15/ENG02/090
COMPUTER ENG

15/ENG02/090
COMPUTER ENG

(c) $y = [(x+1)^2 (x-2)^{1/2}] / [(2x-1)(x+3)^{3/2}]$

(2) $y = [3e^{x} \sin 2x] / [x^{5/2}]$

Integrate the following with respect to the variable

(i) $4 \sec^2 (3m+1)$

(ii) $2t(3t^2 - 1)^{1/2}$

(iii) $2x / (4x^2 - 1)^{1/2}$

Sol

$$y = \ln \frac{(x+1)^2 (x-2)^{1/2}}{(2x-1)(x+3)^{3/2}}$$

$$\ln y = \ln((x+1)^2) + \ln(\sqrt{x-2}) - \ln(2x-1) - \ln[(x+3)^{3/2}]$$

$$(1/y)\frac{dy}{dx} = \frac{1}{(x+1)^2} \cdot 2(x+1) + \frac{1}{\sqrt{x-2}} \cdot \frac{1}{2}(x-2)^{-1/2} - \frac{1}{2x-1} \cdot 2 - \frac{1}{(x+3)^{3/2}} \cdot \frac{3}{2}(x+3)^{1/2}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{2}{(x+1)} + \frac{1}{2(x-2)(\sqrt{x-2})} - \frac{2}{2x-1} - \frac{3}{2}(x+3)^{\frac{1}{2} - \frac{3}{2}}$$

$$\frac{dy}{dx} = y\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{3}{2(x+3)}\right]$$

$$\frac{dy}{dx} = \frac{(x+1)^2 (x-2)^{1/2}}{(2x-1)(x+3)^{3/2}}\left[\frac{2}{x+1} + \frac{1}{2x-4} - \frac{2}{2x-1} - \frac{3}{2x+6}\right]$$

$$y = \frac{3e^{2x}\sin 2x}{x^{5/2}}$$

$$\ln y = \ln(3e^{x}) + \ln(\sin 2x) - \ln(x^{5/2})$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{3e^{x}}\cdot 3e^{x} + \frac{1}{\sin 2x}\cdot 2\cos 2x - \frac{1}{x^{5/2}}\cdot\frac{5}{2}x^{3/2}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = 1 + \frac{2\cos 2x}{\sin 2x} - \frac{5}{2} \times \frac{3}{2} - \frac{5}{2}$$

$$\frac{dy}{dx} = y\left[1 + \frac{2\cos 2x}{\sin 2x} - \frac{5x^{-1}}{2}\right]$$

$$\frac{dy}{dx} = \frac{3e^{x}\sin 2x}{x^{5/2}}\left[1 + \frac{2\cos 2x}{\sin 2x} - \frac{5}{2x}\right]$$

$$\int 4\sec^2(3m+1)\,dm$$

$$4\int \sec^2(3m+1)\,dm$$

let $v = 3m+1$

$$\frac{dv}{dm} = 3$$

$dv = 3dm$, $dm = \frac{dv}{3}$

$$4\int \sec^2(v)\,\frac{dv}{3}$$

$$\frac{4}{3}\int \sec^2(v)\,dv$$

$\frac{4}{3} \tan v + c$

$= \frac{4}{3} \tan(3m+1) + c$

(ii) $\int 2t\,(3t^2 - 1)^{1/2}\,dt$

let $v = \sqrt{3t^2 - 1}$

$v^2 = 3t^2 - 1$

$3t^2 = v^2 + 1$

$t^2 = \frac{v^2+1}{3}$

$t = \sqrt{\frac{v^2+1}{3}}$

$\frac{dt}{dv} = \frac{1}{2}\left(\frac{v^2+1}{3}\right)^{-1/2} \cdot \frac{2v}{3}$

$\frac{dt}{dv} = \frac{v}{3}\left(\frac{v^2+1}{3}\right)^{-1/2}$

$dt = \frac{v\,dv}{3}\left(\frac{v^2+1}{3}\right)^{-\frac{1}{2}}$

$\int 2\left(\frac{v^2+1}{3}\right)^{1/2} \cdot v \cdot \frac{v\,dv}{3}\left(\frac{v^2+1}{3}\right)^{-1/2}$

$\int 2\left(\frac{v^2+1}{3}\right)^{1/2} \cdot v \cdot \frac{v\,dv}{3}\,\frac{(v^2+1)}{3}^{-1/2}$

$= \frac{2}{3}\int v^2 \left(\frac{v^2+1}{3}\right)^{1/2 \cdot -1/2} dw$

$= \frac{2}{3}\int v^2 dv$

$= \frac{2}{3}\left[\frac{v^3}{3}\right] + c$

$= \frac{2v^3}{9} + c$

$= \frac{2(3t^2-1)^{3/2}}{9} + c$

(5). (a) $\int \frac{2x}{\sqrt{4x^2-1}} dx$

soln

let $v = \sqrt{4x^2-1}$

$v^2 = 4x^2-1$

$4x^2 = v^2+1$

$x^2 = \frac{v^2+1}{4}$

$x = \sqrt{\frac{v^2+1}{4}}$

$\frac{dx}{dv} = \frac{1}{2}\left(\frac{v^2+1}{4}\right)^{-1/2} \cdot \frac{v}{2}$

$\frac{dx}{dv} = \frac{v}{4}\left(\frac{v^2+1}{4}\right)^{-1/2}$

$\int \frac{2\left(\frac{v^2+1}{4}\right)^{1/2}}{v} \cdot \frac{v}{4 \cdot 2}\left(\frac{v^2+1}{4}\right)^{-1/2} dv$

$\frac{1}{2}\int \left(\frac{v^2+1}{4}\right)^{1/2 - 1/2} dv$

$\frac{1}{2}\int dv$

$= \frac{v}{2} + c$

$= \frac{4x^2-1}{2} + c$