1.) $3te^{2t}$.

Solution

let $u = 3t$ and $dv = e^{2t}$

$\frac{du}{dt} = 3$ $\int dv = \int e^{2t}$

$\times$

$du = 3dt$ $V = \frac{e^{2t}}{2}$

Using $uv - \int v\,du = \int u\,dv$

$= 3t\left(\frac{e^{2t}}{2}\right) - \int \frac{e^{2t}}{2} \times 3dt$.

$3t\left(\frac{e^{2t}}{2}\right) - \frac{1}{2}\int 3e^{2t}\,dt$

$3t\left(\frac{e^{2t}}{2}\right) - \frac{1}{2} \times \frac{3e^{2t}}{2} + C$

$\int 3te^{2t}$ [illegible] $3e^{2t}] + C$.

2.) $\int x^2 \sin x$

let $u = x^2$ and $dv = \sin x$

$\frac{du}{dx} = 2x$ and $v = -\cos x$

using $uv - \int v\,du$

$(x^2)(-\cos x) - \int(-\cos x)(2x\,dx)$

$-x^2\cos x - \int -2x\cos x\,dx$

$$\left[\begin{array}{l} \text{let } u = -2x \text{ and } du = \cos x \\ du/dx = -2x \text{ and } v = \sin x \end{array}\right]$$

$\therefore (-2x)(\sin x) - \int(\sin x)(-2)\,dx$

$-2x\sin x - (-2)\int \sin x\,dx.$

$-2x\sin x - (-2)\int -\cos x + C.$

$-2x\sin x - 2\cos x + C.$

$\therefore \int x^2 \sin x = -x^2\cos x - 2x\sin x - 2\cos x + C_{11}$

Let $A = 7x$, $B = 2x$

$$\sin 7x \cos 2x = \frac{1}{2}\left[\sin(7x + 2x) + \sin(7x - 2x)\right]$$

$$\sin 7x \cos 2x = \frac{1}{2}\left[\sin 9x + \sin 5x\right]$$

$$= \frac{1}{2}\int\left[\frac{\sin 9x}{9} + \frac{\sin 5x}{5}\right]$$

$$= \cancel{\left(\frac{\sin 9x}{18} + \frac{\sin 5x}{10} + C\right)}$$

$$= -\frac{\cos 9x}{18} - \frac{\cos 5x}{10} + C$$

$$-x^2$$
$$-\underline{(-x^2 + x^3)}$$
$$-x^3$$

which can now be

$$\int (2x - x^2)\,dx + \int \frac{x^3}{1-x}\,dx$$