Q1

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18/EN/GO4/051
MATH 104.
Assignment.

1. Integrate: $4\sec^2(3x+k)$

Differentiate $y = [(x+1)^2(x-2)^{1/2}] / [(2x-1)(x+3)^{3/2}]$.

Using logarithmic rule

$$y = \frac{UV}{WR}$$

$$\ln y = \ln U + \ln V - \ln W - \ln R.$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{U}\frac{dU}{dx} + \frac{1}{V}\frac{dV}{dx} - \frac{1}{W}\frac{dW}{dx} - \frac{1}{R}\frac{dR}{dx}$$

$$\frac{dy}{dx} = y\left[\frac{1}{U}\cdot\frac{dU}{dx} + \frac{1}{V}\frac{dV}{dx} - \frac{1}{W}\frac{dW}{dx} - \frac{1}{R}\frac{dR}{dx}\right].$$

$U = (x+1)^2$, $dU/dx = 2(x+1)$

$V = (x-2)^{1/2}$; $dV/dx = \frac{1}{2}\cdot(x-2)^{-1/2} = \frac{1}{2} \times \frac{1}{(x-2)^{1/2}}$

$W = (2x-1)$, $\frac{dW}{dx} = 2(2x-1)$

$R = (x+3)^{3/2}$; $\frac{dR}{dx} = \frac{3}{2}(x+3)^{1/2}$

$$\frac{dy}{dx} = \frac{[(x+1)^2(x-2)^{1/2}]}{[(2x-1)\cdot(x+3)^{3/2}]}\left[\frac{1}{(x+1)^2}\cdot 2(x+1) + \frac{1}{(x-2)^2}\cdot\frac{1}{2(x-2)^{1/2}} - \frac{1}{(2x-1)}\cdot\frac{1}{2(x-1)} - \frac{1}{(x+3)^{3/2}}\cdot\frac{3}{2}(x+3)\right]$$

$$dy/dx = \frac{[(x+1)^2(x-2)^{1/2}]}{(2x-1)(x+3)^{3/2}}\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{1}{2(2x-1)^2} - \frac{3}{2}\cdot(x+3)^{-1/2}\right]$$

2. Differentiate: $y = [3e^{x} \sin 2x] / [x^{5/2}]$.

Soln

Using logarithmic rule.

$$y = \frac{uv}{w}$$

$$\ln y = \ln u + \ln v - \ln w.$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} - \frac{1}{w}\frac{dw}{dx}$$

$$\frac{dy}{dx} = y\left[\frac{1}{u}\cdot\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} - \frac{1}{w}\frac{dw}{dx}\right].$$

$u = 3e^{x} \Rightarrow \frac{du}{dx} = 3e^{x}$

$v = \sin 2x \quad \Rightarrow \quad dv/dx = -2\cos 2x$

$w = x^{5/2} \quad \Rightarrow \quad dw/dx = 5/2\, x^{3/2}$

$$\frac{dy}{dx} = [3e^{x} \sin 2x] / [x^{5/2}]\left[\frac{1}{3e^{x}} \cdot 3e^{x} + \frac{1}{\sin 2x}\cdot(-2\cos 2x) - \frac{1}{x^{5/2}}\cdot 5/2\, x^{3/2}\right]$$

$$\frac{dy}{dx} = [3e^{x} \sin 2x] / [x^{5/2}]\left[1 + 2\cot 2x - 5/2\, x\right]$$

2. Differentiate $y = [3e^{x}\sin 2x] / [x^{5/2}]$.

Soln

Using logarithmic rule.

$y = \frac{uv}{w}$

$\ln y = \ln u + \ln v - \ln w$.

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} - \frac{1}{w}\frac{dw}{dx}$

$\frac{dy}{dx} = y\left[\frac{1}{u}\cdot\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} - \frac{1}{w}\frac{dw}{dx}\right]$.

$u = 3e^{x} \Rightarrow \frac{du}{dx} = 3e^{x}$

$v = \sin 2x \quad \Rightarrow \quad dv/dx = -2\cos 2x$

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$\frac{dy}{dx} = [3e^{x}\sin 2x] / [x^{5/2}]\left[\frac{1}{3e^{x}}\cdot 3e^{x} + \frac{1}{\sin 2x}\cdot(-2\cos 2x) - \frac{1}{x^{5/2}}\cdot 5/2\, x^{3/2}\right]$

$\frac{dy}{dx} = [3e^{x}\sin 2x] / [x^{5/2}]\left[1 + 2\cot 2x - 5/2\, x\right]$

Q2

1. Integration $4\sec^2(3x+1)$.

$$\int 4\sec^2(3x+1)$$

let $u = 3x+1$, $dx = \frac{1}{3}du$

$$\int 4\sec^2 u \, du$$

$$= \frac{4}{3}\int \sec^2 u \, du$$

$$= \frac{4}{3}\tan u + C$$

$$= \frac{4}{3}\tan(3x+1) + C \text{ Ans.}$$

2. Integrate $2t(3t^2-1)^{1/2}$

$$\int 2t\cdot(3t^2-1)^{1/2}dt$$

Let $u = 3t^2-1$, $\frac{du}{dt} = 6t$, $dt = \frac{1}{6t}du$

$$\int 2t\cdot(3t^2-1)^{x} dt = \int 2t\, u^{1/2}\cdot\frac{1}{6t}du$$

$$= \frac{2}{6}\int u^{1/2}du$$

$$= \frac{2}{3}\cdot\frac{2}{3}u^{1/2} + C$$

$$= \frac{4}{9}u^{1/2} + C$$

$$= \frac{4}{9}(3t^2-1) + C \text{ Ans.}$$

Q2

1. Integration $4\sec^2(3x+1)$.

$\int 4\sec^2(3x+1)$

let $u = 3x+1$, $dx = \frac{1}{3}du$

$\int 4\sec^2 u \, du$

$= \frac{4}{3}\int \sec^2 u \, du$

$= \frac{4}{3}\tan u + C$

$= \frac{4}{3}\tan(3x+1) + C$ Ans.

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let $u = 3t^2-1$, $\frac{du}{dt} = 6t$, $dt = \frac{1}{6t} du$

$\int 2t \cdot (3t^2-1)^{x} dt = \int 2t u^{1/2} \cdot \frac{1}{6t} du$

$= \frac{2}{2}\int u^{1/2} du$

$= \frac{2}{3} \cdot \frac{2}{3} u^{1/2} + C$

$= \frac{4}{9} u^{1/2} + C$

$= \frac{4}{9}(3t^2-1) + C$ Ans.

3 · Integrate $2x / (4x^2 - 1)^{1/2}$

$$\int 2x / (4x^2 - 1)^{1/2}$$

let $u = 4x^2 - 1$, $du = 8x \, dx$, $dx = \frac{1}{8x} du$

$$\int \frac{2x}{(4x^2-1)^{1/2}} dx = \int \frac{2x}{u^{1/2}} \cdot \frac{1}{8x} du$$

$$= \frac{1}{4} \int u^{-1/2} du$$

$$= \frac{1}{4} \left[-1/2 \left[u^{-1/2 + 1} \right] + C \right]$$

$$= -\frac{1}{8} u^{1/2} + C$$

$$= -1/8 \cdot \left[4x^2 - 1 \right]^{1/2} + C //$$

3 · Integrate $2x / (4x^2-1)^{1/2}$

$$\int 2x / (4x^2-1)^{1/2}$$

let $u = 4x^2 - 1$, $\therefore du = 8x\,dx$, $\therefore dx = \frac{1}{8x}\,du$

$$\int \frac{2x}{(4x^2-1)^{1/2}}\,dx = \int \frac{2x}{u^{1/2}} \cdot \frac{1}{8x}\,du$$

$$= \frac{1}{4} \int u^{-1/2}\,du$$

$$= \frac{1}{4} \left[-1/2 \left[u^{-1/2+1} \right] + C \right]$$

$$= -\frac{1}{8}\, u^{1/2} + C$$

$$= -1/8 \cdot \left[4x^2 - 1\right]^{1/2} + C \,//.$$