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18/SCI01/099

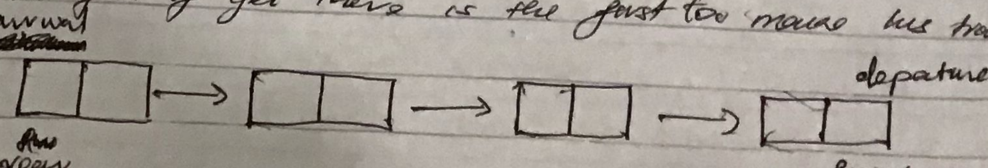
CSC 314

1 Queues or waiting lines stand for a number of customers waiting to be serviced. The process or system that does the services is called the service channel or service facility. An example of a queue is a group of broken down phones, waiting to be fixed by an engineer. The study of waiting line is important because the waiting line is where queues are formed, it is where arrival and departure is showed. It shows how long customers wait on a queue and this information is used to improve waiting lines.

2 Poisson arrival pattern proves that customers arrived in a random fashion and this makes arrival unpredictable because the customer is an independent individual therefore the service provider has no control over the customer. An example is the arrival of broken cars into a mechanic shop, since in its normalty nobody can tell when their car is going to have accidents therefore the arrival of broken cars into a mechanic shop is random

Exponential servies refers to the distribution of the services to customers Eg a single ticket counter where arriving customers will form a queue. In a single channel multiphase an exaple would be a cinema where pop corn is free, therefore after leaving the ticket counter you would go to the popcorn counter to claim your free popcorn

3 Simple queue: This is a type of queue in which ~~arrival~~ arrival occurs at the ~~[illegible]~~ rear of the queue and departure occurs at the front of the queue. An example is a queue at an ATM where the first of get there is the first too make his transaction



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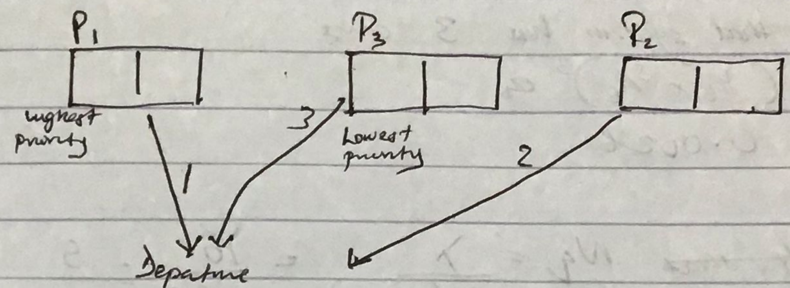
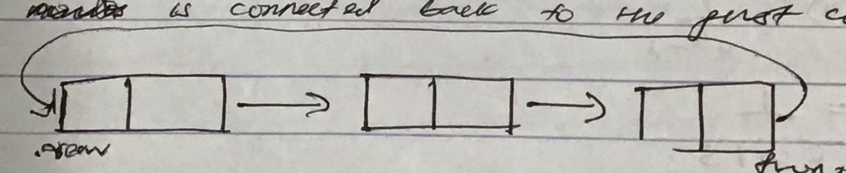
ii) Priority Queue ÷ This contains a queue which have customers with priority levels. The first customer to leave the queue is the customer with the highest priority at a certain moment. an example is in a hospital full of corona virus patients, the patients that would be attended to first will be the ones with severe syntoms while the ones with mild syntome will either be treated later or given less attention.

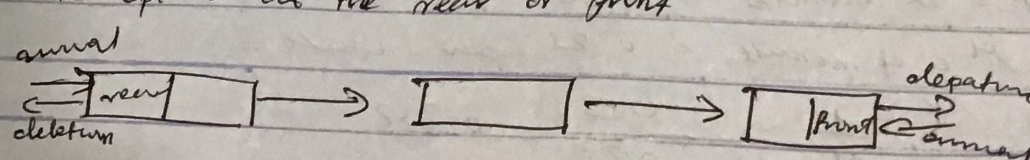


iii) Circular queue

In a circular queue all customers are treated as circular. last ~~queue~~ customers is connected back to the first customer



iv) DE queue ÷ This contains data items which have insert and delete options at the rear or front



4. λ = Ten cars per hour 10/hr

$\mu = \frac{60}{5} = 12$ cars per hour

$\rho = \lambda/\mu = 10/12 = 5/6 < 1$

$P_0 = 1 - \rho = 1/6$

$P_n = (1-\rho)\rho^n$
$= (1/6 \times 5/6)$

a) Probability that the system has two or less cars.
$P_0 + P_1 + P_2 = (1/6) + (1/6 \times 5/6) + (1/6 \times 5/6)^2$
$= 0.3248$

b) Probability that system has 3 cars
$P_3 = (1/6 \times 5/6)^3$
$= 0.0026$

c) ~~Probability that~~ $N_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{10}{12(2)} = \frac{5}{12}$

5) $\lambda = 1/40 = 0.025$
$\mu = 1/25 = 0.04$
$E(4) = \frac{1}{\mu - \lambda}$
$= \frac{1}{0.015}$
$= 66.67$ minutes $\simeq 67$ minutes

6) $\lambda = 12$ minutes $= 0.083$ per min
$\mu = 4$ minutes $= 0.25$ per minute
$\rho = \frac{\lambda}{\mu} = \frac{0.083}{0.25} = 0.332$

a) $P_0 = (1-\rho)$
$= (1 - 0.332)$
$= 0.668 \simeq 0.67$
The probability is 67 %

b)
=
c)
d) P =
e
8 Suero

b) P(waiting time > 10')

$= \int_{0}^{\alpha} (\lambda/\mu)(\mu-\lambda) \times e^{-(\mu-\lambda)t} \, dt$

$= (\lambda/\mu) \times [0 - e^{-(\mu-\lambda)w}]$

$= (\lambda/\mu) \times e^{-(\mu-\lambda)w}$

$= 0.083/0.25 \times e^{-(0.25-0.083)\cdot 10}$

$= 0.06$

c) average length of the queue from time to time

$= \frac{\mu}{\mu-\lambda}$

$= \frac{0.25}{0.25-0.083}$

$= 1.49 \simeq 1.5$

d) $P = (\lambda/\mu)$

$= 0.0332$

e

f Average waiting time in queue $= Gw$

$G = \frac{\lambda}{0.25(0.25-\lambda)}$

$\lambda = 0.3125 - 1.25\lambda$

$\frac{2.25\lambda}{2.25} = \frac{0.3125}{\lambda}$

$= 0.138$

$\simeq 0.14$

7 $\lambda = 4/60$ per minute $= 0.067$

Probability (waiting time) $= 0.90$ (ρ) let $\rho = 0.90$

ρ is waiting time of the customer in the system

$$\rho = \frac{\lambda}{\mu}$$

$$0.9 = \frac{0.067}{\mu}$$

$$\mu = 0.074$$

$$\mu\rho(1-\rho)e^{-\mu w(1-\rho)}$$

$$= 0.074 \times 0.9(1-0.9)e^{-0.074\times 2(1-0.9)}$$

$$= 0.00061$$

$$\lambda = (9/5) = 1.8 \text{ per minute}$$

$$\mu = (10/5) = 2 \text{ per minute}$$

$$\rho = \frac{\lambda}{\mu} = \left(\frac{1.8}{2}\right) = 0.9$$

a) $$E_{(n)} = \frac{\rho}{1-\rho}$$

$$= \frac{0.9}{1-0.9}$$

$= 9$ customers per day

b) $$1 - \left[1 - \rho\left[(1-\rho^{n})/1-\rho\right]\right]$$

$$= 1-(1-\rho^{n})$$

$$= \rho^{n}$$

$$= 0.9^{n}$$

$$= 0.313$$

c $\int_2^\infty (\lambda/\mu)(\mu-\lambda) \times e^{-(\mu-\lambda)t}\, dt$

$= (\lambda/\mu)(\mu-\lambda) \times \left[(e^{-(\mu-\lambda)t}) / -(\mu-\lambda)\right]_2^\infty$

$= (\lambda/\mu) \times e^{-(\mu-\lambda)2}$

$= \frac{1.8}{2} \times e^{-(2-1.8)2}$

$= 0.9 \times e^{-0.4}$

$= 0.60$

$\mu = 12/5 = 2.4$ per min

and $\rho = \frac{\lambda}{\mu} = \frac{1.8}{2.4} = 0.75$

the values for then change to

a) = 3 customers

b) = 0.042

c) = 0.23

9 a) $\lambda = 9/5$; $\mu = 10/5$; $\rho = 9/10$

$L_q = L_s - \rho = \frac{\lambda^2}{\mu(\mu-\lambda)}$

$= \frac{9/5}{10/5(10/5 - 9/5)}$

$= 9/2$

b) $P(N > n) = \rho^{n+1}$

$n = 10$

$P(N > 10) = \rho^{4} = \rho^{4} = (9/10)^4$

$= 0.3138$

c $P(w_q > t) = \lambda/\mu \, e^{-(\mu-\lambda)}$

$P(w_q > 2)$ $4/10$ $e^{-(10/6 - 4/15)}$

$= 0.7368$

10 μ λ $= \frac{60}{20}$

a) average no of plane in queue for good weather $= \frac{400}{60}$ $\frac{10}{40}$

$\lambda^2/\mu \times 1/\mu - \lambda = 1/6$

μ λ $= \frac{30}{20}$

average no of plane in queue for bad weather $= \frac{400}{30} \times \frac{1}{10}$

$=$

b) average no of ~~plane~~ wait time in the system for good weather

$= 1/4$

$1/\mu - \lambda = 1.5$ min

average no of wait time in the system for bad weather

$= 1/20$

$1/\mu - \lambda = 6$ mins

11 $\lambda = 25$ per hr

$\mu = 30$ ~~sets~~ custo per hr

Avg wait time $= \frac{25}{30(30-25)}$

$= 1/6$

$= 0.166 \simeq 0.2$

12 $\lambda = 4$ customers / hr

$\mu = 60/6 = 10$ customers / hr

$\rho = \lambda/\mu = 4/10 = 0.4$

a $P_0 =$

b

c $L_s =$

d w_s

13 $\lambda =$

μ

a

b) L

c w_q

d

14 λ

μ

a

P_0

15 λ

a)

a $P_0 = 1 - \rho = 0.6$

b $1-(1-\rho) = 0.4$

c $L_s = \frac{\lambda}{\mu - \lambda} = \frac{4}{6} = 0.667 \simeq 0.7$

d $W_s = \frac{1}{\mu - \lambda} = 0.1$ mins

13 $\lambda = 2$ belts per shift $\qquad \rho = 2/5$

$\mu = 5$ belts per shift

a fraction of time $P_0 = 1 - \rho = 1 - 2/5 = 0.667 \simeq 0.7$

b $L_s = \lambda/\mu - \lambda = 2/5-2 = 2/3$

c $W_q = \lambda/\mu(\mu-\lambda) = 2/5(3) = 2/15$

d $W_s = 1/\mu - \lambda = 1/5-2 = 1/3$ shift

14 $\lambda = 6 \qquad M_2 = 12/hr$

$\mu - \lambda = 8$

a in case $M\lambda$ is hired

$$er = \frac{1}{m\ b} = \frac{1}{8-6} = \frac{1}{2}\text{ hrs}$$

Total cost of breakdown $= 1/2 \times Rs.20 + 1/8$ hrs Rs 10

$\Rightarrow Rs\ 11.25$

15 $\lambda = 10$ per hr

$\mu = 4$ mins $= 1/4 \times 60 = 15$ per hr

a) $\rho = \lambda/\mu = 0.33$

$\therefore$ 33% is the time a person has to wait b expected percentage of idle time

$= 1 - P$

$= 1 - 0.33$

$= 0.67$

$\therefore$ 67 % is the expected percentage of time.

c expected length of waiting time

$$\frac{\lambda}{\mu(\mu-\lambda)} = \frac{10}{15(15-10)}$$

$= 0.13$

14 $\lambda = 0.2$

$\mu = 1$

$$P_n = 1 \Big/ \sum_{n=0}^{4} [4!/(4-n)!] (0.2/1)^n$$

$= 0.4$

$\therefore$ 40 % of the time the system is empty & busy 60 %

a Expected no of operating machines

$E(n) = M - (\mu/\lambda)(1 - P_0)$

$= 4 - (1/0.2)(1 - 0.4)$

$= 4 - 3$

$= 1$

b Expected downtime cost per day

$= 8 \times 1 \times 25$

$=$ Rs. 200 / Per day

o When there are two mechanics each serving two machines,

$M = 2$

$P_0 = 1 / \sum_{n=0}^{2} [2!(2-n)!] (0.2/1)^n$

$= 1/1 + (2 \times 0.2) + (2 \times 1 \times 0.2^2)$

$= 0.68$

i.e 68 % of the time, the system with no employ

$M - (\mu/\lambda) \times (1+P_0)$

$= 2 - (1/0.2)(1-0.68)$

$= 2 - 1.6$

$= 0.4$

expected downtime per day $= 8 \times 0.4 \times 2$

$= 6.4$ hrs per day

hence cost involved $=$ Rs. $55 \times 2 + 6.4 \times$ Rs. 25

$=$ Rs. 270 per day

~~Total cost~~

15 $4/\lambda \div 4/\mu$

$= \mu/\lambda$

$P/1-P = \dfrac{\mu/\lambda}{1-\mu/\lambda}$

$= \dfrac{\mu/\lambda}{1-\mu/\lambda}$

$= \dfrac{\lambda-\mu}{\lambda}$

$\dfrac{\mu}{\lambda} \times \dfrac{\lambda}{(\lambda-\mu)}$

$\therefore \dfrac{\mu}{\lambda = 2}$

20 $\lambda = 85 \quad = 0.02$ per minute

$\mu = 25 \quad = 0.04$ per minute

$\rho = \frac{\lambda}{\mu} = 0.5$

a) Percent of time barber will be idle

$= 1 - \rho$

$= 1 - 0.5$

$= 0.5$

$\therefore$ 5% of the time

b) avg time a customer spends in shop

$\frac{1}{\mu - \lambda}$

$= \frac{1}{0.04 - 0.02} = 50$ minutes