

$$8) y = 3t^2 \Rightarrow \frac{dy}{dt} = 6t$$

$$x = \frac{1}{t^2} \Rightarrow \frac{dx}{dt} = -2t^{-3}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t}{-2t^{-3}}$$

$$= -3t^4$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos u \times 9x^2$$

$$= 9x^2 \cos u = 9x^2 \cos(3x^3 + 5)$$

$$\text{but } u = 3x^3 + 5$$

$$= 9x^2 \cos(3x^3 + 5)$$

$$9) y = x^2 \cos 2x e^{4x}$$

Take \log_e of both sides

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

Differentiate with respect to x

$$\frac{1}{y} \left(\frac{dy}{dx} \right) = \frac{1}{x^2} (2x) + \frac{1}{\cos 2x} (-2 \sin 2x)$$

+4

To eliminate $\frac{1}{y}$ multiply through by y

$$\frac{dy}{dx} = \frac{2}{x} y - \frac{2 \sin 2x}{\cos 2x} y + 4y$$

$$\frac{dy}{dx} = \frac{2}{x} y - 2 \tan 2x y + 4y$$

$$\text{but } y = x^2 \cos 2x e^{4x}$$

$$\frac{dy}{dx} = x^2 \cos 2x e^{4x} \left(\frac{2}{x} - 2 \tan 2x + 4 \right)$$

$$10) y = \sin(3x^3 + 5)$$

$$y \text{ let } u = 3x^3 + 5 \Rightarrow y = \sin u$$

$$\frac{du}{dx} = 9x^2 \quad \frac{dy}{du} = \cos u$$

$$\begin{aligned} \text{a) } f \circ g(x) &= 2(4x-2)^2 - 5 \\ &= 2(16x^2 - 16x + 4) - 5 \\ &= 32x^2 - 32x + 8 - 5 \\ &= 32x^2 - 32x + 3 \end{aligned}$$

$$\begin{aligned} \text{b) } g \circ f(x) &= 4(2x^2 - 5) - 2 \\ &= 8x^2 - 20 - 2 \\ &= 8x^2 - 22 \end{aligned}$$

$$\begin{aligned} \text{6) } f(x) &= 3x^2 - 2x + 1 = 0 \\ \text{show } f_e(x) + f_o(x) &= f(x) \\ f_e(x) &= \frac{f(x) + f(-x)}{2} \end{aligned}$$

$$\begin{aligned} f(-x) &= 3(-x)^2 - 2(-x) + 1 \\ &= 3x^2 + 2x + 1 \end{aligned}$$

$$\begin{aligned} f_e(x) &= \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2} \\ &= \frac{6x^2 + 2}{2} = 3x^2 + 1 \end{aligned}$$

$$\begin{aligned} f_o(x) &= f(x) - f_e(x) \\ &= \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2} \\ &= \frac{-4x}{2} = -2x \end{aligned}$$

$$\begin{aligned} f(x) &= f_e(x) + f_o(x) \\ &= 3x^2 + 1 + (-2x) \\ &= 3x^2 - 2x + 1 = 0 \end{aligned}$$

7) $y = \cos x \rightarrow$ differentiate

$$y + \Delta y = \cos(x + \Delta x)$$

$$\Delta y = \cos(x + \Delta x) - y \text{ but } y = \cos x$$

$$\Delta y = \cos(x + \Delta x) - \cos x = -\text{①}$$

recall

$$\cos(A+B) - \cos(A-B) = -2\sin A \sin B \text{ -②}$$

comparing ① & ②

$$A+B = x + \Delta x$$

$$A-B = x$$

$$B = \Delta x/2 \text{ \& } A = x + \Delta x/2$$

comparing ① & ②

$$\Delta y = \cos(x + \Delta x) - \cos x = -2\sin A \sin B$$

$$\therefore -2\sin(x + \Delta x/2) \sin(\Delta x/2)$$

$$\text{Divide through by } \Delta x$$

$$\frac{\Delta y}{\Delta x} = -2\sin(x + \Delta x/2) \sin(\Delta x/2)$$

$$\Delta x \qquad \Delta x$$

$$\Delta y = -2\sin(x + \Delta x/2) \sin(\Delta x/2)$$

$$= -\sin(x + \frac{\Delta x}{2}) \times \sin(\frac{\Delta x}{2})$$

Taking limit $\Delta x \rightarrow 0$

$$= \lim_{\Delta x \rightarrow 0} \frac{\sin(\Delta x/2)}{(\Delta x/2)} = 1$$

$$\frac{\Delta y}{\Delta x} = -\sin x + 0 \times 1$$

$$\therefore \Delta y / \Delta x = -\sin x$$

Name: Onyeneke Innocent
 Department: Pharmacy
 Matric no.: 19/MHSU/119

1) $y = \frac{1}{x-2}$

but $y = \frac{1}{2-2} = \frac{1}{0} = \text{undefined}$

∴ The function $y = \frac{1}{x-2}$ is defined for all set of real numbers except $x = 2$

• Domain = all set of real numbers except $x = 2$

• Codomain = all set of real numbers except $y = 0$

2) $h = \ln v$

if $h = \ln v$

$\frac{d}{dx} (\ln v) = \frac{1}{v} \frac{dv}{dx}$

3a) $2x - 3y = 2 = 0$

$\frac{3y}{3} = \frac{2x-2}{3}$

$y = \frac{2x-2}{3}$

$x^2 + y^2 = 4$

$y^2 = -x^2 + 4$

$y = \sqrt{4-x^2}$

Let $P = \sin^{-1} t$, find derivative of P

$P = t$

\sin

$t = \sin P$

$\frac{dt}{dP} = \cos P$ but we need $\frac{dP}{dt} = \frac{1}{\cos P}$

recall

$\cos^2 P + \sin^2 P = 1$

∴ $\cos^2 P = 1 - \sin^2 P$

∴ $\cos P = \sqrt{1 - \sin^2 P}$

but $\sin P = t \Rightarrow \sin^2 P = t^2$

$\cos P = \sqrt{1-t^2}$

hence

$\frac{dP}{dt} = \frac{1}{\cos P} = \frac{1}{\sqrt{1-t^2}}$

5) $f(x) = 2x^2 - 5$

$g(x) = 4x - 2$