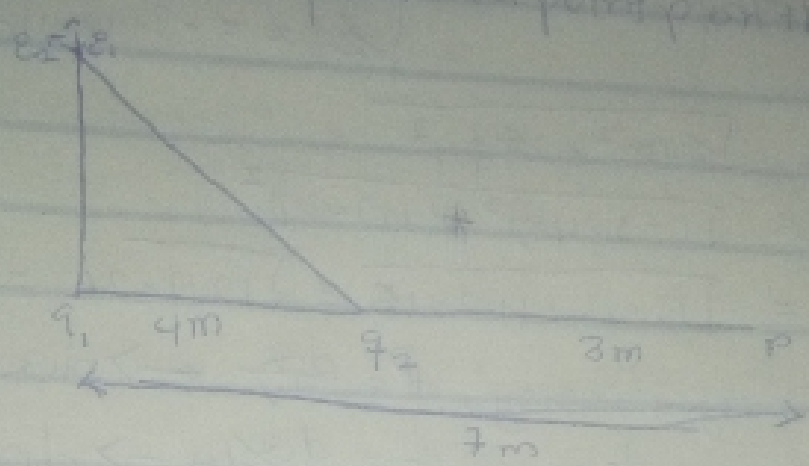


2. Angular Momentum Conservation: Deriving the
 2a Section A

2a Electric field is a region of space in which a charge will experience an electric force whose Electric field intensity is the force per unit charge

- b $q_1 = 8 \mu\text{C}$ at origin, $q_2 = 12 \mu\text{C}$, x -axis $a = 4 \text{ m}$
- c net electric field at a point P on the x -axis at $x = 7 \text{ m}$

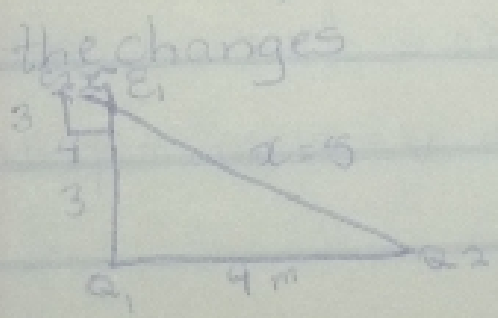


$$E_1 = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{7^2} = 1.5 \text{ N/C}$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-6}}{3^2} = 12 \text{ N/C}$$

$$E_{\text{net}} = E_1 + E_2 = (1.5 + 12) \text{ N/C} = 13.5 \text{ N/C}$$

ii Electric field at a point Q on the y -axis at $y = 3 \text{ m}$



$$a_2^2 = 0^2 + 3^2$$

$$a^2 = 3^2 + 4^2$$

$$a = \sqrt{5} = 5$$

$$E_1 = \frac{kq}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2} = 8 \text{ N/C}$$

$$E_2 = \frac{kq}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{5^2} = 4.32 \text{ N/C}$$

Vector	Angle	x comp	y comp
$E_1 = 8 \text{ N/C}$	90°	0 N/C	8 N/C
$E_2 = 4.32 \text{ N/C}$	36.87°	-3.48 N/C	2.59 N/C
Total		$E_{fx} = -3.48 \text{ N/C}$	$E_{fy} = 10.59 \text{ N/C}$

$$E_{\text{net}} = \sqrt{E_{fx}^2 + E_{fy}^2}$$

$$= \sqrt{(-3.48)^2 + (10.59)^2}$$

$$= \sqrt{12.11 + 112.05} = \sqrt{124.16} = 11.14 \text{ N/C}$$

- 3a) i) Volume charge density, $\rho = \frac{dq}{dv} \rightarrow dq = \rho dv$
 ii) Surface charge density, $\sigma = \frac{dq}{dA} \rightarrow dq = \sigma dA$
 iii) Linear charge density, $\lambda = \frac{dq}{dl} \rightarrow dq = \lambda dl$

b) Electric Potential difference equation. Due to a simple point charge.

$$V_B - V_A = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

where Q = Point charge, r_B = distance Q to B
 r_A = distance Q to A V = electric potential

36 $Q_1 = 10 \mu\text{C}$, $Q_2 = -2 \mu\text{C}$, $x = 0$, $x = 4 \text{ m}$

find position along x -axis where $V = 0$

$$V_p = k \left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2} \right] \quad \left[\begin{array}{c} 4 \\ 4 \text{ m} \end{array} \right] \quad r_2 = x$$

$$= 9 \times 10^9 \left[\frac{10 \times 10^{-6}}{4+x} + \frac{-2 \times 10^{-6}}{x} \right]$$

$$0 = 9 \times 10^9 \times \left[\frac{10 \times 10^{-6}}{4+x} + \frac{-2 \times 10^{-6}}{x} \right]$$

$$0 = \frac{10 \times 10^{-6}}{4+x} + \frac{-2 \times 10^{-6}}{x} = \frac{10 \times 10^{-6}}{4+x} = \frac{2 \times 10^{-6}}{x}$$

$$10 \times 10^{-6} x = (4+x) (2 \times 10^{-6})$$

$$10 \times 10^{-6} x = 8 \times 10^{-6} + 2 \times 10^{-6} x$$

$$10 \times 10^{-6} x - 2 \times 10^{-6} x = 8 \times 10^{-6}$$

$$8 \times 10^{-6} x = 8 \times 10^{-6}$$

$$x = \frac{8 \times 10^{-6}}{8 \times 10^{-6}} = 1 \text{ position along the } x\text{-axis}$$

Section B

4a) Magnetic flux is defined as the strength of the magnetic field which can be represented by lines of force denoted as Φ

b) $m_e = 9.11 \times 10^{-31} \text{ kg}$, $r = 1.04 \times 10^{-7} \text{ m}$, $B = 1.06 \times 10^{-19}$, cyclotron frequency = ω

$$10 = qvB = \frac{m_e v^2}{r^2}$$

$$m_e v = q B r \Rightarrow v = \frac{q B r}{m_e} = \frac{1.6 \times 10^{-19} \times 3 \times 10^{-2} \times 10^{-2}}{9.11 \times 10^{-31}}$$

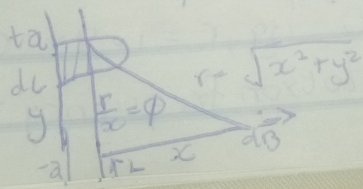
$$e = 6.14 \times 10^{10} \text{ C}^{-1} \rightarrow \text{Cyclotron frequency}$$

c) In 4b we were given $m_e = 9.11 \times 10^{-31} \text{ kg}$, $r = 10^{-2} \text{ m}$, $B = 3.5 \times 10^{-1} \text{ W/m}^2$ and asked to find the cyclotron frequency also known as angular speed. It is called cyclotron frequency because it is a frequency of an accelerator called cyclotron.

Recall $\omega = \text{angular speed} = \text{cyclotron frequency} = \frac{q v B}{m_e}$
It has a unit of $\frac{1}{\text{s}}$ which is the unit of frequency dimensionally.

5a) Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space (μ_0), the current (I), the change in length, the radius and inversely proportional to square of radius (r^2).

b) Magnetic field of a straight current carrying conductor



A section of a straight current carrying conductor

Applying Biot-Savart law for (infinitesimal) wire of length dl field (dB) is

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \theta}{r^2}$$

$$\sin(\pi - \theta) = \sin \theta$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{r^2}$$

from the diagram $r^2 = x^2 + y^2$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{x^2 + y^2} \quad \dots \quad (1)$$

$$\text{But } \sin(\pi - \theta) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}}$$

Substitute (1) into (1)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

$$dl = dy \Rightarrow B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy$$

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{\sqrt{x^2 + y^2}} \cdot \frac{1}{x^2} \cdot \frac{y}{\sqrt{x^2 + y^2}}$$

$$B = \frac{\mu_0 I}{4\pi x} \left[\frac{2a}{(x^2 + a^2)^{1/2}} \right] \therefore (x^2 + a^2)^{1/2}$$

$$B = \frac{\mu_0 I}{2\pi x} = \frac{\mu_0 I}{2\pi r}$$