

$$b) y = \frac{(3e^x \sin 2x)}{(x^{5/2})}$$

differentiating both sides of the equation

$$\frac{d}{dx} (y) = \frac{d}{dx} \left(\frac{3e^x \sin(2x)}{x^{5/2}} \right)$$

For the right side,

$$\frac{12e^x x^{3/2} \cos(2x) + 6e^x x^{5/2} \sin(2x) - 15e^x x^{3/2} \sin(2x)}{2x^5}$$

For right side,

$$= 3 \frac{d}{dx} \left(\frac{e^x \sin(2x)}{x^{5/2}} \right)$$

using quotient rule, $f(x) = e^x \sin 2x$
 $g(x) = x^{5/2}$

$$= 3 \frac{x^{5/2} \frac{d}{dx} [e^x \sin(2x)] - e^x \sin(2x) \frac{d}{dx} [x^{5/2}]}{(x^{5/2})^2}$$

multiplying the exponents in $(x^{5/2})^2$

$$= 3 \frac{x^5 \frac{d}{dx} [e^x \sin(2x)] - e^x \sin(2x) \frac{d}{dx} [x^{5/2}]}{x^5}$$

using product rule,
 $3 - e^x \sin(2x) \frac{d}{dx} [x^{5/2}]$

$$= 3 \frac{-e^x \sin(2x) \left(\frac{5}{2} x^{3/2} - 1 \cdot \frac{1}{2} \right)}{x^5}$$

$$= 3 \frac{-e^x \sin(2x) \left(\frac{5}{2} x^{3/2} + \frac{-1 \cdot 1}{2} \right)}{x^5}$$

$$= 3 \frac{-e^x \sin(2x) \left(\frac{5}{2} x^{3/2} \right)}{x^5}$$

$$= 3 \left(\frac{2x^{5/2} (2e^x \cos(2x)) + \sin(2x) 2x e^x - 5 \sin(2x) x^{3/2} e^x}{2x^5} \right)$$

$$= 3 \left(\frac{2x^{5/2} (2e^x \cos(2x)) + \sin(2x) 2x e^x - 5 \sin(2x) x^{3/2} e^x}{2x^5} \right)$$

$$= \frac{12e^x x^{5/2} \cos(2x) + 6e^x x^{3/2} \sin(2x) - 15e^x x^{3/2} \sin(2x)}{2x^5}$$

simplifying,

$$= \frac{12e^x x^{5/2} \cos(2x) + 6e^x x^{3/2} \sin(2x) - 15e^x x^{3/2} \sin(2x)}{2x^5}$$

By setting the left side equal to the right,

$$\frac{12e^x x^{5/2} \cos(2x) + 6e^x x^{3/2} \sin(2x) - 15e^x x^{3/2} \sin(2x)}{2x^5}$$

Answer

$$= \frac{12e^x x^{5/2} \cos(2x) + 6e^x x^{3/2} \sin(2x) - 15e^x x^{3/2} \sin(2x)}{2x^5} //$$

a) $2x(4x^2-1)^{1/2}$
 $\int \frac{2x}{(4x^2-1)^{1/2}} dx$
 $2 \int \frac{x}{(4x^2-1)^{1/2}} dx$
 $u = 4x^2 - 1, \frac{du}{dx} = 8x$
 $\frac{1}{8} du = x dx$
 $2 \int \frac{1}{u^{1/2}} \cdot \frac{1}{8} du$
 $2 \int \frac{1}{8u^{1/2}} du$
 $2 \left(\frac{1}{8} \int \frac{1}{u^{1/2}} du \right)$
 $= \frac{1}{4} \int u^{-1/2} du$
 $\frac{1}{4} \int 4^{-1/2} du$
 $\frac{1}{4} (2u^{1/2} + C)$
 $= \frac{u^{1/2}}{2} + C$
 $= \frac{(4x^2-1)^{1/2}}{2} + C$
 $= \frac{1}{2} (4x^2-1)^{1/2} + C$

b) $\frac{2t}{(3t^2-1)^{1/2}}$
 $\int \frac{2t}{(3t^2-1)^{1/2}} dt$
 $2 \int \frac{t}{(3t^2-1)^{1/2}} dt$
 let $u = 3t^2 - 1, \frac{1}{6} du = t dt$
 $\frac{du}{dt} = 6t$
 $= 2 \int \frac{1}{u^{1/2}} \cdot \frac{1}{6} du$
 $= 2 \int \frac{1}{6u^{1/2}} du$
 $= 2 \left(\frac{1}{6} \int \frac{1}{u^{1/2}} du \right)$
 $= \frac{1}{3} \int u^{-1/2} du$
 $= \frac{1}{3} (2u^{1/2} + C)$
 $= \frac{2u^{1/2}}{3} + C$
 $= \frac{2}{3} (3t^2-1)^{1/2} + C$

a) $4 \sec^2(3m+1)$
 $4 \int \sec^2(3m+1) dm$
 let $u = 3m + 1$
 $\frac{du}{dm} = 3$
 so, $\frac{1}{3} du = dm$
 $= 4 \int \sec^2(u) \frac{1}{3} du$
 $= 4 \int \frac{\sec^2(u)}{3} du$
 $= 4 \left(\frac{1}{3} \int \sec^2(u) du \right)$
 $= \frac{4}{3} \int \sec^2(u) du$
 $= \frac{4}{3} (\tan(u) + C)$
 $= \frac{4}{3} \tan(u) + C$
 $= \frac{4}{3} \tan(3m+1) + C$