Math Submit assignment

1. Differentiate the following

1. $y=\frac{[(x+1)^2(x-2)^{1/2}]}{[(2x-1)(x+3)^{3/2}]}$

2. $y=\frac{[3e^{x}\sin 2x]}{(x^{5/2})}$

Integrate the following with respect to the variable.

3. $4\sec^2(3m+1)$

4. $2t(3t^2-1)^{1/2}$

5. $\frac{2x}{\sqrt{(4x^2-1}}$

Answers.

1. $y=\frac{[(x+1)^2(x-2)^{1/2}]}{[(2x-1)(x+3)^{3/2}]}$

$\ln y = [\ln(x+1)^2+\ln(x-2)^{1/2}]-[\ln(2x-1)+\ln(x+3)^{3/2}]$

$\frac{1}{y}\cdot\frac{dy}{dx}=\left[\frac{1}{(x+1)^2}\cdot 2(x+1)+\frac{1}{(x-2)^{1/2}}\cdot\frac{1}{2}(x-2)^{-1/2}\right]-\left[\frac{1}{2x-1}\cdot 2+\frac{3}{2}(x+3)^{1/2}\cdot\frac{1}{(x+3)}\right]$

$\frac{dy}{dx}=y\left[\frac{2(x+1)}{(x+1)^2}+\frac{(x-2)^{-1/2}}{2(x-2)^{1/2}}\right]-\left[\frac{2}{2x-1}+\frac{3(x+3)^{1/2}}{2(x+3)^{3/2}}\right]$

$\frac{dy}{dx}=y\left[\frac{2}{(x+1)}+\frac{1}{2}(x-2)^{-1}\right]-\left[\frac{2}{2x-1}+\frac{3}{2}(x+3)^{-1}\right]$

$\frac{dy}{dx}=\frac{(x+1)^2(x-2)^{1/2}}{((2x-1)(x+3)^{3/2})}\left(\frac{2}{(x+1)}+\frac{1}{2(x-2)}-\frac{2}{2x-1}-\frac{3}{2(x+3)}\right)$

2. $y=\frac{(3e^{x}\sin 2x)}{x^{5/2}}$

$\ln y = \ln 3e^{x}+\ln\sin 2x-\ln(x^{5/2})$.

$\frac{1}{y}\frac{dy}{dx}=\frac{1}{3e^{x}}\cdot 3e^{x}+\frac{1}{\sin 2x}2\cos 2x-\frac{1}{x^{5/2}}\cdot\frac{5}{2}x^{3/2}$

$\frac{dy}{dx}=y\left(\frac{3e^{x}}{3e^{x}}+\frac{2\cos 2x}{\sin 2x}-\frac{5x^{3/2}}{2x^{5/2}}\right)$

$\frac{dy}{dx}=y\left(1+\frac{2\cos 2x}{\sin 2x}-\frac{5}{2}x^{-1}\right)$

$\frac{dy}{dx}=\frac{3e^{x}\sin 2x}{x^{5/2}}\left(1+\frac{2\cos 2x}{\sin 2x}-\frac{5x^{-1}}{2}\right.$

3. $\int 4\sec^2(3m+1)\,dm$

$4\int \sec^2(3m+1)\,dm$

let $u = 3m+1$

$\frac{du}{dm} = 3$

$dm = \frac{du}{3}$

$4\int \sec^2 u \; du/3$

$4/3 \int \sec^2 u$

$= \frac{4}{3}\tan u + c$

$= \frac{4}{3}\tan(3m+1) + c$

4. $\int 2t(3t^2-1)^{1/2}\,dt$

$u = (3t^2-1)^{1/2}$

$u = \sqrt{3t^2-1}$

$u^2 = 3t^2-1$

$3t^2 = u^2+1$

$t = \sqrt{\frac{u^2+1}{3}} = \cancel{\left(\frac{u^2+1}{3}\right)^{1/2}}$

let $u = 3t^2-1$

$\frac{du}{dt} = 6t$

$dt = \frac{du}{6t}$

$\int 2t \cdot u^{1/2}\frac{du}{6t}$

$\int \frac{1}{3}\sqrt{u}\,du$

$\frac{1}{3}\int u^{1/2}\,du$

$\frac{1}{3}\left(\frac{u^{1/2+1}}{3/2}\right) + c$

$\frac{1}{3}\left(\frac{2u^{3/2}}{3}\right) + c$

$= \frac{2u^{3/2}}{9} + c$

$= \frac{2(3t^2-1)^{3/2}}{9} + c$

5. $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

let $u = 4x^2-1$

$\frac{du}{dx} = 8x$

$dx = \frac{du}{8x}$

$\int \frac{2x}{\sqrt{u}} \cdot \frac{1}{8x}\,du$

$\int \frac{1}{4}\frac{1}{\sqrt{u}}\,du$

$\frac{1}{4}\int \frac{1}{\sqrt{u}}\,du$

$\frac{1}{4}\left(\frac{u^{-1/2+1}}{-1/2}\right) + c$

$\frac{1}{4}\left(\frac{u^{1/2}}{1/2}\right) + c$

$\frac{1}{4}\left(2\sqrt{u}\right) + c$

$\frac{\sqrt{u}}{2} + c$

$\frac{\sqrt{4x^2-1}}{2} + c$