Maths 104

1. Differentiate $y = \sin\left(\frac{6}{x^2}\right)$ from first principle.

$$y = \sin\left(\frac{6}{x^2}\right)$$

$$y + \Delta y = \sin\left(\frac{6}{(x+\Delta x)^2}\right)$$

$$\Delta y = \sin\left[\frac{6}{(x+\Delta x)^2}\right] - \sin\left(\frac{6}{x^2}\right)$$

Recall $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$

$$\Delta y = 2\cos\left(\frac{12x^2 + 12x\Delta x + 6\Delta x^2}{2x^2(x+\Delta x)^2}\right) \cdot \sin\left(\frac{12x\Delta x - 6\Delta x^2}{2x^2(x+\Delta x)^2}\right)$$

~~divide by co-efficient Δ~~

$$\Delta y = 2\cos\left[\frac{12x^2 + 12x\Delta x + 6\Delta x^2}{2x^2(x+\Delta x)^2}\right] \cdot \sin\left(\frac{12x\Delta x - 6\Delta x^2}{2x^2(x+\Delta x)^2}\right) \times \frac{-6}{x^3}$$

$$-6\Delta x$$

$$\frac{\Delta y}{\Delta x} = \frac{-12\cos}{x^3}\left[\frac{12x^2 + 12x\Delta x + 6\Delta x^2}{2x^2(x+\Delta x)^2}\right] \cdot \sin\left(\frac{12x\Delta x - 6\Delta x^2}{2x^2(x+\Delta x)^2}\right)$$

$$\lim_{\Delta x \to 0} \frac{dy}{dx} = \lim_{\Delta x \to 0}\left(\frac{12x^2 + 12x\Delta x + 6\Delta x^2}{2x^2(x^2+\Delta x)^2}\right) \lim_{\Delta x \to 0}\left(\frac{12x\Delta x - 6\Delta x^2}{2x^2(x+\Delta x)^2}\right)$$

$$\frac{dy}{dx} = \frac{-12\cos}{x^3}\left(\frac{12x^2}{2x^2(x^2)}\right) \cdot 1$$

$$\frac{dy}{dx} = -\frac{12\cos 6x^{-2}}{x^3}$$

2. $x = 4t^3 - t^2$ $\quad y = t^4 + 2t^2$ at $t = 3$ and 1

Using $A = \int_a^b y\,dx$

but $x = 4t^3 - t^2$

$$\frac{dx}{dt} = 12t^2 - 2t$$

$$dx = (12t^2 - 2t)\,dt$$

$$A = \int_1^3 (t^4 + 2t^2) \cdot (12t^2 - 2t)\, dt$$

$$A = \int_1^3 12t^6 - 2t^5 + 24t^4 - 4t^3\, dt$$

$$A = \left[\frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{4t^4}{4} \right]_1^3$$

$$A = \left[\frac{12t^7}{7} - \frac{t^6}{3} + \frac{24t^5}{5} - t^4 \right]_1^3$$

$$A = \left[\frac{12(3)^7}{7} - \frac{(3)^6}{3} + \frac{24(3)^5}{5} - 3^4 \right] - \left[\frac{12(1)^7}{7} - \frac{1(1)^6}{3} + \frac{24(1)^5}{5} - 1^4 \right]$$

$$A = \left[\frac{26244}{7} - 243 + \frac{5832}{5} - 81 \right] - \left[\frac{12}{7} - \frac{1}{3} + \frac{24}{5} - 1 \right]$$

$$A = \frac{160704}{35} - \frac{544}{105}$$

$A = 4586.36$ sq. units.

5. $x = 4t^3 - t^2$, $y = t^4 + 2t^2$. find dy/dx

using $\cancel{\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}}$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\frac{dy}{dt} = 4t^3 + 4t \qquad \frac{dx}{dt} = 12t^2 - 2t$$

$$\frac{dy}{dx} = \frac{4t^3 + 4t}{12t^2 - 2t} = \frac{2t(2t^2 + 2)}{2t(6t - 1)}$$

$$\therefore \frac{dy}{dx} = \frac{2t^2 + 2}{6t - 1}.$$