

Question 1 (Couette flow)

- One of the plates must be stationary while the other tends to infinity.
- The flow must be laminar between the plates.
- acceleration and pressure gradient are zero.

Question 2 (Determination of the nature of flow)

- through the determination of its Reynold's number
- presence of local acceleration
- ~~also~~ whether the flow is rotational

Aerofoil	Hydrofoil
- Applies to air as medium in concerned	- Applies to liquids (water) as medium in concern.

Question 1

$$\mu = 0.9 \text{ Centipoise}$$

$$b = 10 \text{ mm} = 0.01 \text{ m}$$

$$y = ?$$

$$U = 1 \text{ ms}^{-1}$$

$$dx = 60 \text{ mm}$$

$$dp = 60 \text{ kN/m}^2 = 60 \times 10^3 \text{ N/m}^2$$

(i) Velocity distribution

$$U = \frac{Uy}{b} - \left[\frac{1}{2\mu} \left(\frac{dp}{dx} \right) (by - y^2) \right]$$

$$U = \frac{1 \times y}{0.01} - \left[\frac{1}{2 \times (0.9 \times 10^{-3})} \cdot \left(\frac{-60 \times 10^3 \text{ N/m}^2}{60 \text{ mm}} \right) \cdot (0.01y - y^2) \right]$$
$$= 100y - [0.556 \times 10^3 \times -10^3 \times (0.01y - y^2)]$$

$$= 100y - [-5560y + (5.56 \times 10^5 y^2)]$$
$$= 5660y + (5.56y^2 \times 10^5)$$
$$= \text{same} + \text{same}$$

(ii) Flow Rate $q = \frac{Uy}{2} - \frac{b^3}{12\mu} \left(\frac{dp}{dx} \right)$

$$= \frac{1 \times 0.01}{2} - \left[\frac{(0.01)^3}{12(0.9 \times 10^{-3})} \cdot \left(\frac{-60 \times 10^3}{60} \right) \right]$$
$$= 0.005 - [9.259 \times 10^{-5} \times -10^3]$$
$$= 0.005 + 9.259 \times 10^{-2}$$
$$= 0.005 + 0.0926$$
$$= 0.0976 \text{ m}^3/\text{s}$$

(iii) Shear Stress distribution where $y = b$

$$= \frac{\mu U}{b} - \left[\frac{1}{2} \left(\frac{dp}{dx} \right) \cdot (b - 2y) \right]$$

$$= \frac{0.9(10^{-3})}{0.01} - \left[\frac{1}{2} [-10^3] \cdot (0.01 - 2y) \right]$$

$$= 0.09 - [0.5 \times -10^3 \times (0.01 - 2y)]$$

$$= 0.09 - [-5 + 10^3 y]$$

$$= 5.09 + 1000y = -4.91 \text{ N/m}^2$$

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Question 2

$$\mu = 0.9 \text{ Ns/m}^2$$

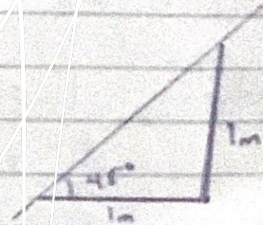
$$\rho = 1260 \text{ kg/m}^3$$

$$U = -1.5 \text{ m/s}^{-1}$$

$$b = 0.01 \text{ m}$$

$$P_1 = 250 \text{ kNm}^{-2}$$

$$P_2 = 80 \text{ kNm}^{-2}$$



$$\sin 45^\circ = 1/x$$

$$x = 1/\sin 45^\circ$$

$$x = \sqrt{2}$$

Solution

$$P_1' = P_1 + \rho g h$$

$$= 250 \times 10^3 + (1260 \times 9.81 \times 1)$$

$$= 262.6 \times 10^3 \text{ Nm}^{-2}$$

$$P_2' = P_2 + \rho g h$$

$$= 80 \times 10^3 + (1260 \times 9.81 \times 0)$$

$$= 80 \times 10^3$$

$$\Delta P = 262.6 \times 10^3 - 80 \times 10^3$$

$$= 182.6 \times 10^3 \text{ Nm}^{-2}$$

Flow in downslope while upper plate moves upslope

$$-\frac{\Delta P}{\Delta x} = \left(\frac{182.6 \times 10^3}{\sqrt{2}} \right)$$

$$\frac{\Delta P}{\Delta x} = -128.95 \times 10^3 \text{ Nm}^{-3}$$

(i) Velocity distribution

$$U = \frac{U_y}{b} - \left[\frac{1}{2\mu} \cdot \left(\frac{\Delta P}{\Delta x} \right) \cdot (by - y^2) \right]$$

$$= \frac{-1.5 y}{0.01} - \left[\frac{1}{2(0.9)} \cdot (-128.95 \times 10^3) (0.01y - y^2) \right]$$

$$= -150y - [0.556 \cdot (-128.95 \times 10^3) (0.01y - y^2)]$$

$$= -150y - [-716.96y - 7.17 \times 10^4 y^2]$$

$$= -150y + 716.96y + 7.17 \times 10^4 y^2$$

$$= +566.96y + (7.17 \times 10^4) y^2$$

(ii) Stress (shear) distribution

$$\tau_{xy} = \mu \frac{du}{dy}$$

where

$$u = 566y + 7.17 \times 10^4 y^2$$

$$\frac{du}{dy} = 566 + 1.43 \times 10^5 y$$

$$\tau = \mu \left(\frac{du}{dy} \right)$$

$$\tau = 0.9(566 + 1.43 \times 10^5 y)$$

$$\tau = 509.4 + 1.287 \times 10^5 y$$

(iii) Max Velocity

U_{\max} occurs at $du/dy = 0$

Substituting back into the previous equation

$$0 = 566.4 + 1.43 \times 10^5 (y)$$

$$y = \frac{-566.4}{1.43 \times 10^5}$$

$$y = -3.958 \times 10^{-3} \text{ m}$$

Therefore

$$U_{\max} = 566.4(y) + (7.17 \times 10^4)(y^2)$$

$$= -2.24 + 1.12$$

$$= -1.12 \text{ ms}^{-1}$$

In the opposite direction of flow

$$(iii) \quad \tau = 509.4 + 1.287 \times 10^5 (-3.958 \times 10^{-3})$$

$$= 509.4 + (-509.39)$$

$$= 0.01 \text{ Nm}^{-2}$$