

15) i) Length is at most 2

$$L = \{ \epsilon, a, b, ab, aa, ba, bb \}$$

$$(a+b)^* (a+b)^*$$

ii) Language of even length

$$L = \{ aa, bb, aabb, aaabbb, \dots \}$$

$$(a+b)(a+b)^*$$

iii) Language starting and ending with the same letter

$$a(a+b)^*a$$

iv) Language starting and ending with different letters

$$a(a+b)^*b + b(a+b)^*a$$

16) Let L be a language over an alphabet, then L is a regular language if and only if there exists a regular expression R over an alphabet such that $L = L(R)$. A regular expression is a sequence of characters that define a search pattern.

Identities of R.G

i) $\emptyset + R = R$

ii) $\emptyset R + R\emptyset = \emptyset$

iii) $\epsilon R = R\epsilon = R$

iv) $\epsilon^* = \epsilon$ and $\emptyset^* = \epsilon$

v) $R + R = R$

vi) $R^* R^{\#} = R^{\#}$

vii) $R R^{\#} = R^* R$

$$x_{11}) (P+Q)R = PR + QR \text{ and } R(P+Q) = RP + RQ$$

- 17) Let A be a non empty alphabet the expression r and its corresponding language $L(r)$ are defined inductively as follows. The symbol λ and parentheses $()$ are regular expressions, each letter a in A is a regular expression. If r is a regular expression r^* is a regular expression if r_1 and r_2 are regular then r_1r_2 is a R.E. If r_1 and r_2 are R.E then r_1, r_2 are R.E.
- ii) The symbols used in regular expression are $\{ (), ^*, \cup, \lambda, /, \}^*$

18) i) $r = b^*$

The language $L(r)$ consists of all b 's including ϵ
e.g $\{ \epsilon, b, bb, bbb, \dots \}$

ii) Let $r = aa^*$

The language $L(r)$ consists of all positive powers of a excluding the empty word.

iii) Let $r = a \cup b^*$

The language $L(r)$ consists of a or any word in b .

iv) Let $r = (a \cup b)^*$

The language $L(r)$ consists of all words over the given alphabet A

vii) Let $\Sigma = \{a, b\}^*$

The language $L(\Sigma)$ consists of all combinations of a & b

19) $\Sigma = \{a, b\}$

$$L_1 = \{b^m a^n \mid m > 0, n > 1\}$$

$$L_2 = \{a^n b^m a \mid m > 1, n > 0\}$$

$$L_3 = \{a b^n \mid n > 0\}$$

Find a R.E. over $A = \{a, b\} \Delta L$

$$L_i = L(\Sigma) \quad \forall i = 1, 2, 3$$

$$L_1 = b b^* a a a^*$$

$$L_2 = a a^* b b b b^* a$$

$$L_3 = a b b^*$$

20) A regular set is any set represented by regular expression
eg $a, b, \epsilon \Sigma$ then .

a denotes the set $\{a\}$

$a b$ denotes the set $\{a, b\}$

$a b$ denotes $\{ab\}$

a^* denotes the set $\{\epsilon, a, aa, aaa, \dots\}$

$(a+b)^*$ denotes $\{\{a, b\}^*\}$

The set represented by R is denoted by $L(R)$. Eg
Let R_1 and R_2 denote any two regular expression
Then

from R_1

3 A string in $L(R^*)$ is a string obtained by concatenating n elements for some $n \geq 0$

Q1) $\{110\}$

$\{1\}$ and $\{0\}$ are represented by 1 and 0 respectively therefore 110 is obtained by concatenating 1, 1 and 0

ii) $\{baab\}$

$\{b\}$ and $\{a\}$ are represented by b and a respectively therefore baab is obtained by concatenating b, a, a and b

iii) $\{01, 10\}$

as $\{01, 10\}$ is the union of $\{01\}$ and $\{10\}$ then we have it represented as 01 + 10

iv) $\{\epsilon, 10\}$

The set $\{\epsilon, 10\}$ is also represented by $\epsilon + 10$

v) $\{abb, a, b, bb\}$

The set $\{abb, a, b, bb\}$ is also represented by
 $abb + a + b + bb$

vi) $\{\epsilon, a, aa, aaa, \dots\}$

R.E for this set is $\epsilon + (a)^*$

22) L_1 is the set of all strings of a's and b's ending in aa

$$(a + b)^* aa$$

ii) L_2 is the set of all strings of 0's and 1's beginning with 1 and ending with 0

$$1(1+0)^* 0$$

iii) L_3 is the set of {epsilon, 11, 111, 1111, ...}

$$\text{epsilon } 1(1)^*$$

23) Grammars are finite sets of rules used to describe languages. "Grammar is a generator of language while Language is a set of strings generated by grammar"

24) The sentential derivation of a string is any string derivable from the start symbol S by during language generation. e.g. Consider the grammar $G = (\{S\}, \{a, b\}, S, P)$ with Production rules $S \rightarrow aSb, S \rightarrow \lambda$

$$S \rightarrow aSb \rightarrow ab$$

This is a sentential derivation.