

INAIIBO E. TSATH

16/EN/04/026

Electrical Electronic Engineering

EE 561 - Process Control and Automation Assignment.

Questions

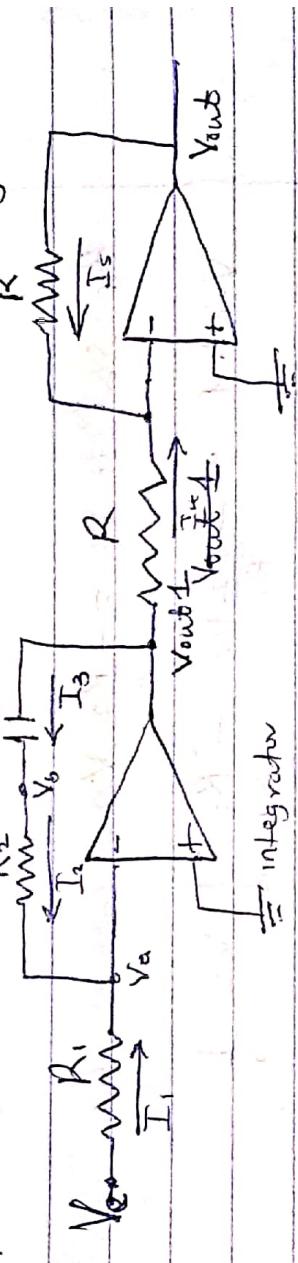
Derive the analysis for the output voltage in using Operational Amplifier for:

Proportional Integral Controller Mode.

Proportional Derivative Mode.

Solutions

Proportional Integral Controller Mode R Design.



$$V_{out} = -R \frac{(V_{out} - 1)}{R} = -V_{out} + 1 \quad [\text{Inverting Circuit}]$$

There is no current flow into the op-amp input terminals as well as voltage across the input terminals. Hence:
 $V_a = 0$.

By applying KCL to the integrator op amp:

$$I_1 + I_2 = 0 \quad \text{--- ①}$$

$$I_3 - I_2 = 0 \quad \text{--- ②}$$

$$\text{Note: } I_2 = I_C = C \frac{dV_C}{dt} = C \frac{d(V_{out} - V_b)}{dt} \quad \text{--- ③}$$

By applying ohm's law to eqn (1) & (2): eqn (1) & (2) below

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \quad \text{--- ④}$$

$$\therefore V_b = -\frac{R_2}{R_1} V_e \quad \text{--- ⑤}$$

$$C \frac{d(V_{out} - V_b)}{dt} = 0 \quad \text{--- ⑥}$$

By expansion; eqn (i) becomes

$$C \frac{dV_{out1}}{dt} = \frac{C dV_b}{dt} - \frac{V_b}{R_2 C} = 0$$

$$\text{Divide through by } C \\ \frac{dV_{out1}}{dt} - \frac{V_b}{R_2 C} = 0 \quad \text{--- (ii)}$$

Take the Laplace of eqn (ii).

$$sV_{out1} - sV_b - \frac{V_b}{R_2 C} = 0$$

$$sV_{out1} = sV_b + \frac{V_b}{R_2 C}$$

$$V_{out1} = V_b + \frac{V_b}{R_2 C} \cdot \frac{1}{s} \quad (\text{divide through by } s).$$

By substituting $V_b = -\frac{R_2}{R_1} V_e$, eqn (ii) becomes:

$$V_{out1} = -\frac{R_2}{R_1} V_e - \frac{R_2}{R_1} \cdot \frac{1}{R_2 C} V_e \cdot \frac{1}{s}$$

$$\text{Also, } V_{out1} = -V_{out2},$$

$$\text{Hence, } -V_{out1} = -\frac{R_2}{R_1} V_e - \frac{R_2}{R_1} \cdot \frac{1}{R_2 C} V_e \cdot \frac{1}{s};$$

$$\therefore V_{out1} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot \frac{1}{R_2 C} V_e \cdot \frac{1}{s}$$

By taking the inverse Laplace transform;

$\frac{1}{s} = \int t dt + K$, $K = \text{constant of integration.}$

and applying the integrator circuit analysis principle;

$\therefore V_{out1} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot \frac{1}{R_2 C} \int V_e dt + V_{out}(0)$

or

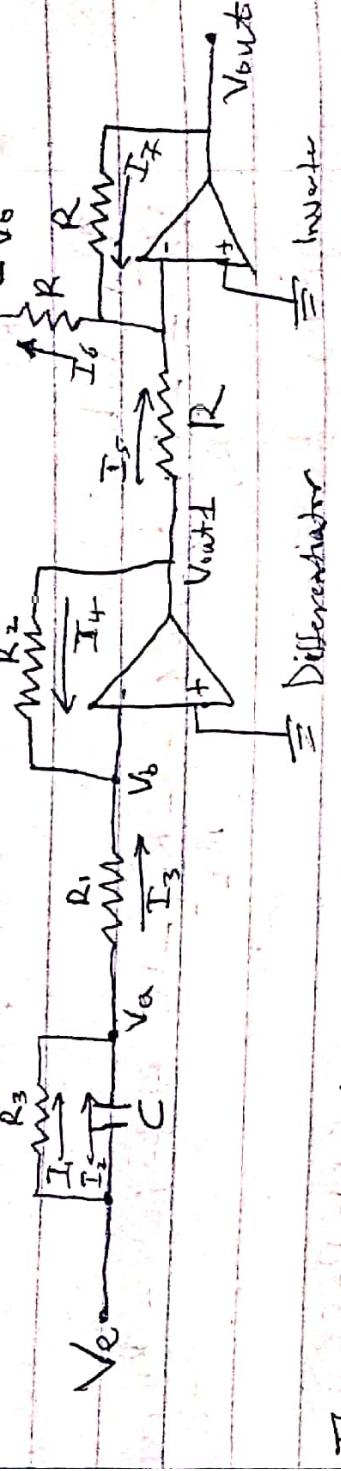
$$V_{out1} = G_p V_e + G_p G_I \int_0^t V_e dt + V_{out}(0).$$

where: $V_{out}(0) = \text{output voltage with no error voltage}$

$$G_p = \frac{R_2}{R_1} = \text{proportional gain}$$

$$GI = \frac{1}{R_2 C} = \text{Integral gain.}$$

Proportional Differentiator Controller Mode Design - V_o



This circuit already includes the clamp to protect against high gain at high frequencies in the derivative term.

The circuit is modified form of the previous derivative mode circuit:

$$R = \frac{R_1 R_3}{R_1 + R_3} (R - \text{Effective Resistance})$$

Then, the condition becomes as usual: $2\pi f_{\text{max}} R C = 0.1$

By KCL

$$I_1 + I_2 - I_3 = 0 \quad \text{(1)}$$

$$I_4 + I_3 = 0 \quad \text{(2)}$$

Combining with ohms law:

$$\frac{V_e - V_a}{R_3} + C \frac{d(V_e - V_a)}{dt} - \frac{V_a}{R_1} = 0 \quad \text{(3)}$$

$$\frac{V_{out1}}{R_2} + \frac{V_a}{R_1} = 0 \quad \text{--- (4)}$$

$$\therefore V_a = - \frac{R_1}{R_2} V_{out1} \quad \text{--- (5)}$$

$$\text{Also: } I_3 + I_4 - I_5 = 0 \quad \text{--- (6)}$$

$$\frac{V_{out1}}{R_2} + \frac{V_{out1} - V_o}{R} = 0 \quad \text{--- (7)}$$

$$\therefore V_{out1} = -V_{out1} + V_o \quad \text{--- (8)}$$

By expansion, eqn (3) becomes:

$$\frac{V_e}{R_3} + \frac{C dV_e}{dt} = \frac{V_a}{R_3} + \frac{V_a}{R_1} + C \frac{dV_a}{dt} = -V_o$$

Take the Laplace off (ignoring)

$$\frac{V_e}{R_3} + sV_e = V_a/R_3 + \frac{V_a}{R_1} + sC \frac{dV_a}{dt}$$

$$V_e \left(\frac{1}{R_3} + SC \right) = V_o \left(\frac{1}{R_3} + \frac{1}{R_1} + SC \right) - \text{---(16)}$$

- By substitution;

$$V_o = -\frac{R_1}{R_2} V_{out} \text{ i.e. } = -\frac{R_1}{R_2} (-V_{out} + V_o) = \frac{R_1}{R_2} (V_{out} - V_o).$$

equation becomes:

$$\left(\frac{R_1}{R_2} \right) (V_{out} - V_o) \left(\frac{1}{R_3} + \frac{1}{R_1} + SC \right) = V_e \left(\frac{1 + R_3 SC}{R_3} \right)$$

$$\left(\frac{R_1}{R_2} \right) (V_{out} - V_o) \left(\frac{R_1 + R_3 + R_1 R_3 SC}{R_3 R_1} \right) = V_e \left(\frac{1 + R_3 SC}{R_3} \right)$$

$$V_{out} - V_o = V_e \left(1 + R_3 SC \right) \times \frac{R_2}{R_1 + R_3 + R_1 R_3 SC}$$

Divide both numerator & Den. by $R_1 + R_3$.

$$V_{out} - V_o = V_e \left(\frac{1}{R_1 + R_3} + \frac{R_3 SC}{R_1 + R_3} \right) \frac{R_2}{R_2 + R_3 SC}$$

$$V_{out} - V_o = \left(\frac{R_2}{R_1 + R_3} + \frac{R_1 R_3 SC}{R_1 + R_3} \right) \frac{V_e}{1 + R_3 SC}$$

If $R_3 SC < < 1$,

$$V_{out} - V_o = \frac{R_2}{R_1 + R_3} V_e + \frac{R_2}{R_1 + R_3} \cdot R_3 C S V_e$$

By taking the inverse Laplace transform;

$$SV_e = \frac{d}{dt} V_e$$

$$V_{out} = \frac{R_2}{R_1 + R_3} V_e + \frac{R_2}{R_1 + R_3} \cdot R_3 C \frac{dV_e}{dt} + V_o$$

or

$$V_{out} = G_p V_e + G_p \frac{dV_e}{dt} + V_o$$

where: $G_p = \frac{R_2}{R_1 + R_3}$ - - Proportional gain.

$G_D = R_3 C$ - - Derivative gain