Name : Okakuro Brume

Mat No: 16/ENG04/041

DEPT : Electrical/Electronics Eng

COURSE : Process Control & Automation

Solution

1 Proportional Integral Controller

$V_a = 0$

$I_1 + I_2 = 0$ ——— (1)

$I_3 - I_2 = 0$ ——— (2)

Current through the Capacitor

$$I_c = C\frac{dV_a}{dt}$$

$$I_1 = \frac{V_e - V_a}{R_1} \quad (V_a = 0)$$

$$I_1 = \frac{V_e}{R_1}$$

$$I_2 = \frac{V_b - V_a}{R_2} \quad (V_a = 0)$$

$$I_2 = \frac{V_b}{R_2}$$

$$I_3 = \frac{C\,d(V_{out1} - V_b)}{dt}$$

Sub into equ (1) & equ (2)

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \text{ ——— } (3)$$

$$\frac{C\,d(V_{out1} - V_b)}{dt} - \frac{V_b}{R_2} = 0 \text{ ——— } (4)$$

from equ (3)

$$\frac{V_b}{R_2} = \frac{-V_e}{R_1}$$

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$$V_b = -\frac{R_2}{R_1} V_e$$

taking the Laplace transform of equ 4

$$sC(V_{out1}(s) - V_b(s)) - \frac{V_b(s)}{R_2} = 0$$

$$sC\,V_{out1}(s) = sC\,V_b(s) + \frac{V_b(s)}{R_2}$$

$$sC\,V_{out1}(s) = V_b(s)\left(sC + \frac{1}{R_2}\right)$$

recall; $V_b = -\frac{R_2}{R_1} V_e$

$$sC\,V_{out1}(s) = -\frac{R_2}{R_1} V_e(s)\left(sC + \frac{1}{R_2}\right)$$

$$V_{out1}(s) = \frac{-R_2}{sCR_1} V_e(s)\left(sC + \frac{1}{R_2}\right)$$

$$V_{out1}(s) = \frac{-R_2}{R_1} V_e(s) + \left(\frac{-R_2}{R_1}\right)\left(\frac{1}{sCR_2} V_e(s)\right)$$

from the Inverting Circuit 5

$$V_{out1} = -V_{out}$$

$$V_{out}(s) = -\left(\frac{-R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{sCR_2} V_e(s)\right)$$

$$V_{out}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{sCR_2} V_e(s)$$

taking the Inverse Laplace

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \times \frac{1}{R_2} \int_0^t V_e(t)\,dt + V(0)$$

$$\left[\text{Where } \mathcal{L}^{-1}\{1/s\} = c\int_0^t dt + K\right]$$

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e\,dt + V(0)$$

Where $G_P = \frac{R_2}{R_1}$ \$ $G_I = \frac{1}{R_2 C}$

2. Proportional Derivative Controller

$$I_1 + I_2 = I_3 \quad \text{—— (1)}$$

$$I_3 + I_4 = 0 \quad \text{—— (2)}$$

$$I_1 = \frac{V_e - V_a}{R_3}$$

$$I_2 = \frac{C\,d\,(V_e - V_a)}{dt}$$

$$I_3 = \frac{V_a - V_b}{R_1} \quad (V_b = 0)$$

$$I_3 = \frac{V_a}{R_1}$$

$$I_4 = \frac{V_{out1} - V_b}{R_2} \quad (V_b = 0)$$

$$I_4 = \frac{V_{out1}}{R_2}$$

$$R = \frac{R_1 R_3}{R_1 + R_3} \quad \text{—— effective resistance}$$

Sub into equ (1) & equ (2)

$$\frac{V_e - V_a}{R_3} + \frac{C\,d\,(V_e - V_a)}{dt} = \frac{V_a}{R_1} \quad \text{—— (3)}$$

$$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0 \quad \text{—— (4)}$$

from equ (4)

$$\frac{V_a}{R_1} = -\frac{V_{out1}}{R_2}$$

$$\boxed{V_{a(t)} = \frac{-R_1}{R_2} V_{out1(t)}}$$

rearranging equ (3)

$$\frac{V_e - V_a}{R_3} + \frac{C\,d}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$$

Taking Laplace transform

$$\frac{V_B(s) - V_a(s)}{R_3} + sC(V_B(s) - V_a(s)) - \frac{V_a(s)}{R_1} = 0$$

(Initial Conditions go to zero)

$$\cancel{\frac{V_B(s) - V_a(s)}{R_3}} \quad \frac{V_B(s)}{R_3} + sCV_B(s) = \frac{V_a(s)}{R_1} + \frac{V_a(s)}{R_3} + sCV_a(s)$$

$$V_B(s)\left(\frac{1}{R_3} + sC\right) = V_a(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right)$$

recall, $V_a = \frac{-R_1}{R_2} V_{out1}$

$$V_B(s)\left(\frac{1}{R_3} + sC\right) = -R_1/R_2 \, V_{out1}(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right)$$

taking the L.C.M

$$V_B(s)\left(\frac{1 + R_3 sC}{\cancel{R_3}}\right) = \frac{-\cancel{R_1}}{R_2} V_{out1}(s)\left(\frac{R_3 + R_1 + sCR_1R_3}{\cancel{R_1}\cancel{R_3}}\right)$$

$$V_B(s)(1 + R_3 sC) = -\frac{V_{out1}(s)}{R_2}(R_3 + R_1 + sCR_1R_3)$$

$$-V_{out1}(s) = \frac{V_B(s)(1 + sCR_3)R_2}{(R_1 + R_3 + sCR_1R_3)}$$

$$-V_{out1}(s) = \frac{V_B(s)(R_2 + sCR_2R_3)}{(R_1 + R_3 + sCR_1R_3)}$$

divide num & den by $R_1 + R_3$

$$-V_{out1}(s) = \frac{V_B(s)(R_2 + sCR_2R_3)/R_1 + R_3}{\frac{\cancel{R_1} + R_3}{R_1 + R_3} + \frac{sCR_1R_3}{R_1 + R_3}}$$

recall, $R = \frac{R_1R_3}{R_1 + R_3}$

$$-V_{out1}(s) = \frac{V_B(s)(R_2 + sCR_2R_3)/R_1 + R_3}{1 + sCR}$$

If $sCR << 1$

$$-V_{out1}(s) = \frac{V_B(s)(R_2 + sCR_2R_3)}{R_1 + R_3}$$

from the Inverting Circuit

$V_{out1} = -V_{out} + V_0$

$$\therefore -(-V_{out} + V_0) = \frac{V_B(s)(R_2 + sCR_2R_3)}{R_1 + R_3}$$

$$V_{out}(s) - V_c(0) = \frac{V_\theta(s) R_2}{R_1+R_3} + \frac{SCR_2R_3}{R_1+R_3} V_\theta(s)$$

$$V_{out}(s) = \frac{R_2}{R_1+R_3} V_\theta(s) + \frac{R_2}{R_1+R_3} R_3 C S V_\theta(s) + V(0)$$

taking inverse Laplace

$$V_{out} = \frac{R_2}{R_1+R_3} V_\theta + \frac{R_2}{R_1+R_3} R_3 C \frac{dV_\theta}{dt} + V(0)$$

$$V_{out} = G_p V_\theta + G_p G_D \frac{dV_\theta}{dt} + V(0)$$

Where; $G_p = \frac{R_2}{R_1+R_3}$

$G_D = R_3 C$