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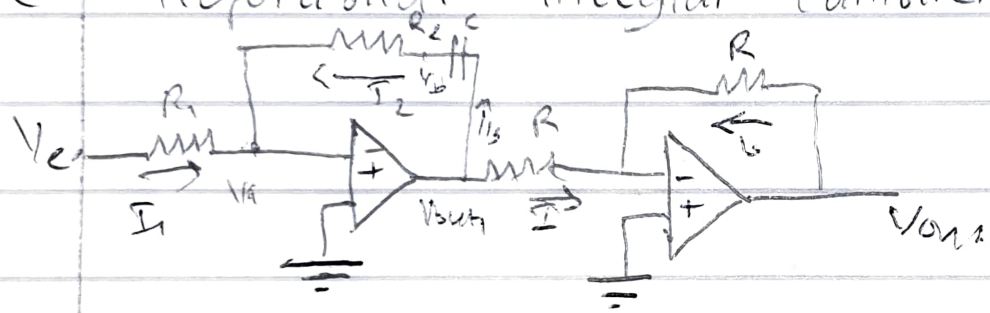
16/ENG04/018

Electrical/Electronics Engineering

EEE 561 [ASSIGNMENT]

Derive the analysis for the output voltage in using operational amplifier for

(1) Proportional integral Controller mode



$$V_{out} = -\frac{R}{R}(V_{out_1}) = -V_{out_1} \quad \text{[Inverting Circuit]}$$

P_i Controller $V_a = 0$

$$I_1 + I_2 = 0 \quad \text{--- (1)}$$

$$I_3 - I_2 = 0 \quad \text{--- (2)}$$

Current through the capacitor

$$I_c = \frac{C dV_c}{dt}$$

Since $V_a = 0$

$$I_1 = \frac{V_e - V_a}{R_1} = \frac{V_e}{R_1}$$

$$I_2 = \frac{V_b - V_a}{R_2} = \frac{V_b}{R_2}$$

$$I_3 = \frac{Cd}{dt}(V_{out_1} - V_b)$$

from eqn (1) $\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0$

$\therefore V_b = -\frac{R_2}{R_1} V_e$

from eqn 2 $\frac{C d(V_{out1} - V_b)}{dt} - \frac{V_b}{R_2} = 0$ -- (3)

taking the laplace transform of eqn (3) above

$sC(V_{out1}(s) - V_b(s)) - \frac{V_b(s)}{R_2} = 0$

$sC V_{out1}(s) = sC V_b(s) + \frac{V_b(s)}{R_2}$

$sC V_{out1}(s) = V_b(s)\left(sC + \frac{1}{R_2}\right)$

$V_b = -\frac{R_2}{R_1} V_e$

$\therefore sC V_{out1}(s) = -\frac{R_2}{R_1} V_e(s)\left(sC + \frac{1}{R_2}\right)$

divide by sC

$V_{out}(s) = \frac{-R_2 V_e(s)}{sCR_1}\left(sC + \frac{1}{R_2}\right)$

(opening the brackets)

$V_{out1}(s) = -\frac{R_2}{R_1} V_e(s) + -\frac{R_2}{R_1}\frac{1}{sCR_2} V_e(s)$

from the following circuit

$V_{out}(s) = -V_{out}$

$$\therefore V_{out} = -\left(-\frac{R_2}{R_1} (V_{e(s)}) - \frac{R_2}{R_1} \frac{1}{sCR_2} V_{e(s)} \right)$$

$$V_{out}(s) = \frac{R_2}{R_1} V_{e(s)} + \frac{R_2}{R_1} \frac{1}{sCR_2} V_{e}(s)$$

taking the inverse laplace

$$V_{out} = \frac{R_2}{R_1} V_{e}(t) + \frac{R_2}{R_1} \frac{1}{R_2 C} \int_0^t V_e(t)\, dt + V_{out}(0)$$

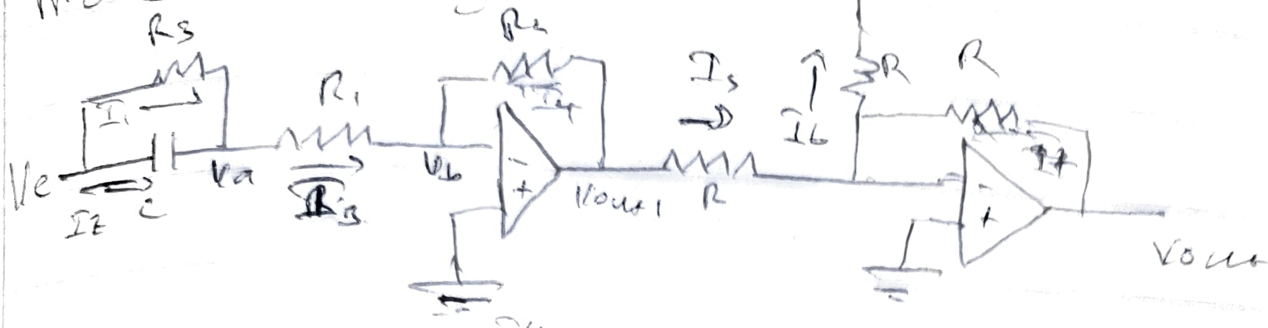
$$\left(\text{where } \frac{1}{s} = \int_0^t dt + K, \quad K = \text{Constant of integration} \right)$$

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e dt + V_{out}(0)$$

$$\text{Where } G_P = \frac{R_2}{R_1} \Rightarrow (\text{Proportional gain})$$

$$G_I = \frac{1}{R_2 C} \Rightarrow (\text{Integral gain})$$

Proportional Derivative Controller mode Design



PD Controller (Taking KCL)

$$I_1 + I_2 = I_3 \quad \text{--- (1)}$$

$$I_3 + I_4 = 0 \quad \text{--- (2)}$$

from eqn (1)

$$\frac{V_e - V_a}{R_3} + \frac{C\,d(V_e - V_a)}{dt} = \frac{V_a}{R_1} \quad \text{--- (3)}$$

from eqn (2)

$$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0 \quad \text{--- (4)}$$

$$V_a = -\frac{R_1}{R_2} V_{out1}$$

from eqn (3)

$$\frac{V_e - V_a}{R_3} + \frac{C\,d(V_e - V_a)}{dt} - \frac{V_a}{R_1} = 0$$

tau

$$I_1 = \frac{V_e - V_a}{R_3}$$

$$I_2 = \frac{C\,d(V_e - V_a)}{dt}$$

$$I_3 = \frac{V_a - V_b}{R_1} \quad (V_b = 0)$$

$$I_3 = \frac{V_a}{R_1}$$

$$I_4 = \frac{V_{out1} - V_b}{R_2}$$

$(V_b = 0)$

$$I_4 = \frac{V_{out1}}{R_2}$$

Effective resistance

$$R = \frac{R_1 R_3}{R_1 + R_3}$$

taking laplace transform of eqn (3)

$$\frac{Ve - Va(s)}{R_3} + sC(Ve(s) - Va(s)) - \frac{Va(s)}{R_1} = 0$$

$$\therefore \frac{Ve}{R_3} - \frac{Va}{R_3} + sCVe(s) - Va(s)sC - \frac{Va(s)}{R_1} = 0$$

rearranging

$$\frac{Ve}{R_3} + sCVe(s) = \frac{Va(s)}{R_1} + \frac{Va(s)}{R_3} + sCVa(s)$$

$$Ve(s)\left(\frac{1}{R_3} + sC\right) = Va(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right)$$

recall $Va = -\frac{R_1}{R_2} Vout$

$$Ve(s)\left(\frac{1}{R_3} + sC\right) = -\frac{R_1}{R_2} Vout\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right)$$

$$Ve(s)\left(\frac{1 + R_3sC}{R_3}\right) = -\frac{\cancel{R_1}}{R_2} Vout\left(\frac{R_3 + R_1 + sCR_1R_3}{\cancel{R_1}R_3}\right)$$

multiply through by R_3

$$Ve(s)\left(\frac{1 + R_3sC}{\cancel{R_3}}\right) \times \cancel{R_3} = -\frac{1}{R_2} Vout\left(\frac{R_3 + R_1 + sCR_1R_3}{\cancel{R_3}}\right) \times \cancel{R_3}$$

$$Ve(s)(1 + R_3sC) = -\frac{Vout(s)}{R_2}(R_3 + R_1 + sCR_1R_3)$$

making V_{out} subject

$$-V_{out}(s) = \frac{V_e(s)(1+sCR_3)R_1}{R_1+R_3+sCR_1R_3}$$

$$-V_{out}(1)(s) = \frac{V_e(s)(R_2+sCR_2R_3)}{R_1+R_3+sCR_1R_3}$$

diving ~~the~~ ~~numerator &~~ through by R_1+R_3

$$V_{out}(s) = \frac{\frac{V_e(s)(R_2+sCR_2R_3)}{R_1+R_3}}{\frac{R_1+R_3}{R_1+R_3}+\frac{sCR_1R_3}{R_1+R_3}}$$

effective resistance $R = \frac{R_1R_3}{R_1+R_3}$

$$-V_{out}(1)(s) = \frac{\frac{V_e(s)(R_2+sCR_2R_3)}{R_1+R_3}}{1+sCR}$$

if $sCR \ll 1$

$$-V_{out}(s) = \frac{V_e(s)\,R_2+sCR_2R_3)}{R_1+R_3}$$

but from inverting (linear)

$$V_{out}1 = -V_{out} + V_c$$

$$-(-V_{out}(s)+V(s)) = \frac{V_e(R_2+sCR_2R_3)}{R_1+R_3}$$

$$-\left(V_- - V_{out}(s) + V(0)\right) = \frac{V_e\left(R_2 + sCR_2R_1\right)}{R_1+R_3}$$

$$V_{out}(s) - V(0) = \frac{V_e(s)\,R_2}{R_1+R_3} + \frac{sCR_2R_1}{R_1+R_3}V_e(s)$$

$$V_{out}(s) = \frac{R_2}{R_1+R_3}V_e(s) + \frac{R_2}{R_1+R_3}R_3\left(CsV_e(s)\right) + V(0)$$

taking the inverse laplace transform

$$V_{out} = \frac{R_2}{R_1+R_3}V_e + \frac{R_2}{R_1+R_3}R_3C\frac{dV_e}{dt} + V_0$$

$$V_{out} = G_pV_e + G_pG_D\frac{dV_e}{dt} + V_0$$

where $G_p = \frac{R_2}{R_1+R_3}$ --- proportional gain

$G_D = R_3C$ --- Derivative gain