NAME: AMOO OLUWAFERANMI MARK

MATRIC NO. : 18/ENG04/077

DEPARTMENT : ELECTRICAL AND ELECTRONICS ENGINEERING

LEVEL : 400 L

COURSE CODE : EEE 441

Course Title: Servo Mechanism & Control Systems

ASSIGNMENT

Question

1. Briefly explain the Root Locus Technique

   Answer.
   The **root locus technique in control system** was first introduced in the year 1948 by Evans. Any physical system is represented by a transfer function in the form of

$$G(s) = k \times \frac{numerator\ of\ s}{denomerator\ of\ s}$$

We can find poles and zeros from G(s). The location of poles and zeros are crucial keeping view stability, relative stability, transient response and error analysis. When the system is put to service stray inductance and capacitance get into the system, thus changes the location of poles and zeros. In **root locus technique in control system** we will evaluate the position of the roots, their locus of movement and associated information. These information will be used to comment upon the system performance.
Now before I introduce what is a root locus technique, it is very essential here to discuss a few of the advantages of this technique over other stability criteria. Some of the advantages of root locus technique are written below.

Advantages of Root Locus Technique

1. Root locus technique in control system is easy to implement as compared to other methods.
2. With the help of root locus we can easily predict the performance of the whole system.
3. Root locus provides the better way to indicate the parameters.
   Now there are various terms related to root locus technique that we will use frequently in this article.

1. Characteristic Equation Related to Root Locus Technique : 1 + G(s)H(s) = 0 is known as characteristic equation. Now on differentiating the characteristic equation and on equating dk/ds equals to zero, we can get break away points.
2. Break away Points : Suppose two root loci which start from pole and moves in opposite direction collide with each other such that after collision they start moving in different directions in the symmetrical way. Or the breakaway points at which multiple roots of the characteristic equation 1 + G(s)H(s) = 0 occur. The value of K is maximum at the points where the branches of root loci break away. Break away points may be real, imaginary or complex.

3. Break in Point : Condition of break in to be there on the plot is written below : Root locus must be present between two adjacent zeros on the real axis.
4. Centre of Gravity : It is also known centroid and is defined as the point on the plot from where all the asymptotes start. Mathematically, it is calculated by the difference of summation of poles and zeros in the transfer function when divided by the difference of total number of poles and total number of zeros. Centre of gravity is always real and it is denoted by σ_A.

$$\sigma_A = \frac{(Sum\ of\ real\ parts\ of\ poles) - (Sum\ of\ real\ parts\ of\ zeros)}{N - M}$$

   Where, N is number of poles and M is number of zeros.
5. Asymptotes of Root Loci : Asymptote originates from the center of gravity or centroid and goes to infinity at definite some angle. Asymptotes provide direction to the root locus when they depart break away points.
6. Angle of Asymptotes : Asymptotes makes some angle with the real axis and this angle can be calculated from the given formula,

$$Angle\ of\ asymptotes\ = \frac{(2p + 1) \times 180}{N - M}$$

   Where, p = 0, 1, 2 ……. (N-M-1)
   N is the total number of poles
   M is the total number of zeros.
7. Angle of Arrival or Departure : We calculate angle of departure when there exists complex poles in the system. Angle of departure can be calculated as 180-{(sum of angles to a complex pole from the other poles)-(sum of angle to a complex pole from the zeros)}.
8. Intersection of Root Locus with the Imaginary Axis : In order to find out the point of intersection root locus with imaginary axis, we have to use Routh Hurwitz criterion. First, we find the auxiliary equation then the corresponding value of K will give the value of the point of intersection.
9. Gain Margin : We define gain margin by which the design value of the gain factor can be multiplied before the system becomes unstable. Mathematically it is given by the formula

$$Gain\ margin = \frac{Value\ of\ K\ at\ the\ imaginary\ axes\ cross\ over}{Design\ value\ of\ K}$$

10. Phase Margin : Phase margin can be calculated from the given formula:

$$Phase\ margin = 180 + \angle(G(jw)H(jw))$$

11. Symmetry of Root Locus : Root locus is symmetric about the x axis or the real axis.

How to determine the value of K at any point on the root loci? Now there are two ways of determining the value of K, each way is described below.

a. Magnitude Criteria : At any points on the root locus we can apply magnitude criteria as,

$$|G(s)H(s)| = 1$$

   Using this formula we can calculate the value of K at any desired point.

b. Using Root Locus Plot : The value of K at any s on the root locus is given by

$$K = \frac{product\ of\ all\ of\ the\ vector\ lengths\ drawn\ from\ the\ poles\ of\ G(s)H(s)\ to\ s}{product\ of\ all\ of\ the\ vector\ lengths\ drawn\ from\ the\ zeros\ of\ G(s)H(s)\ to\ s}$$

2. Describe the use of Routh Hurwitz to find the stability of a closed loop system when:

a. entire row is zero on the Routh table

1. Create an auxiliary polynomial from the row above the row of zeros, skipping every other power of

2. Differentiate the auxiliary polynomial w.r.t.

3. Replace the zero row with the coefficients of the resulting polynomial

4. Complete the Routh table as usual

5. Evaluate the sign of the first-column entries

B. to determine the poles on the jw axis.

1. Form a new polynomial using the entries in the row above zeros. The polynomial will start with power of s in that row, and continue by skipping every other power of s, i.e. P(s) = s 4 + 6s 2 + 8 (5) 2.
2. Next we differentiate the polynomial with respect to s and obtain dP(s) ds = 4s 3 + 12s + 0 (6)
3. Finally the row with all zeros in the Routh table is replaced with the coefficients in Eq.(6), and continue the table. s 5 1 6 8 s 4 67 → 1 642 → 6 656 → 8 s 3 60 →64 → 1 60 →612 → 3 60 → 0 s 2 3 8 0 s 1 1 3 0 0 s 0 8 0 0
4. We see no sign changes hence no rhp poles.
5. Why does an entire row of zeros occur? When a purely odd or even polynomial is a factor of the original polynomial. (s 4 +6s 2 +8 is an even polynomial as it only has even power of s.) σ j ω j ω j ω σ quadrantal and symmetrical symmetrical and imaginary symmetrical and real σ
6. Some polynomial only have roots symmetrical about the origin.
7. Routh table from the even polynomial (s 4 → s 0) is a test of the even polynomial.
8. The rows of zeros indicates the possilbility of jω roots.