ABBEY FLOURISH OBARI-AKASE

16/ENG04/001

ELECT/ELECT

EEE 561

Assignment

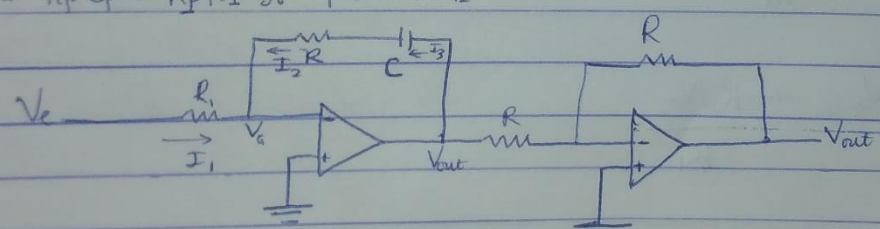
Derive the analysis for the output voltage in using operational Amplifier for:

1. Proportional Integral Controller mode
2. Proportional Derivative Controller mode.

SOLUTION

1. Proportional-Integral mode [PI]

$$P = K_p e_p + K_p K_I \int_0^t e_p \, dt + P_I(0)$$



$V_a = 0$

$I_1 + I_2 = 0$

$I_3 - I_2 = 0$

$$I_c = C \frac{dv}{dt}$$

$$I_1 = \frac{V_e}{R_1}$$

$$I_2 = \frac{V_b}{R_2}$$

$$\Rightarrow I_1 + I_2 = 0$$

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \quad \text{.................} \; (1)$$

$$\Rightarrow \quad I_3 - I_2 = 0$$

$$\Rightarrow I_3 - I_2 = C\frac{d}{dt}\left[V_{out} - V_b\right] - \frac{V_b}{R_2} = 0 \quad \text{......} \; (2)$$

from equation 1

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0$$

$$V_b = -\frac{R_2 V_e}{R_1} \quad \text{..........} \; (3)$$

method 1:

taking laplace transform of equation 2

$$Sc\left[V_{out1(s)} - V_{b(s)}\right] - \frac{V_{b(s)}}{R} = 0$$

$$Sc\,V_{out1(s)} = ScV_{b(s)} + \frac{V_{b(s)}}{R_2}$$

$$ScV_{out1(s)} = V_{b(s)}\left(Sc + \frac{1}{R_2}\right)$$

Equation 3 $\frac{o}{s}$

$$V_b = -\frac{R_2}{R_1}V_e$$

$$\therefore ScV_{out1(s)} = -\frac{R_2}{R_1}V_{e(s)}\left[Sc + \frac{1}{R_2}\right]$$

$$\therefore V_{out1(s)} = -\frac{R_2}{ScR_1}V_{e(s)}\left[Sc + \frac{1}{R_2}\right]$$

$$V_{out1(s)} = -\frac{R_2}{R_1}V_{e(s)} - \frac{R_2}{R_1}\frac{1}{ScR_2}V_{e(s)}$$

from the inverting circuit.

$$V_{out1} = -V_{out}$$

$$\therefore V_{out(s)} = -\left[-\frac{R_2}{R_1}V_{e(s)} - \frac{R_2}{R_1}\frac{1}{ScR_2}V_{e(s)}\right]$$

$$V_{out(s)} = \frac{R_2}{R_1}V_{e(s)} + \frac{R_2}{R_1}\frac{1}{ScR_2}V_{e(s)}$$

Taking inverse laplace

$$V_{out} = \frac{R_2}{R_1} V_{e(s)} + \frac{R_2}{R_1} \frac{1}{R_2} \int_0^t V_{e(t)}\, dt + V_{(0)}$$

or

$$V_{out} = G_p V_e + G_p G_I \int_0^t V_e\, dt + V_{(0)}$$

where

$$G_p = \frac{R_2}{R_1}$$

$$G_I = \frac{r}{R_2 C}$$

method 2:

Substituting equation 3 into 2

$$C \frac{d}{dt}\left[V_{out1} + \frac{R_2 V_e}{R_1} \right] + \frac{R_2 V_e}{R_1 R_2}$$

$$C \frac{dV_{out1}}{dt} + \frac{R_2}{R_1} C \frac{dV_e}{dt} + \frac{V_e}{R_1} = 0$$

Dividing through by C

$$\frac{dV_{out1}}{dt} + \frac{R_2}{R_1} \frac{dV_e}{dt} + \frac{V_e}{R_1 C} = 0 \quad \text{------} \; (4)$$

Integrating equation f with respect to dt

$$\int_0^{V_{out}} \frac{dV_{out1}}{\cancel{dt}} \cdot dt + \frac{R_2}{R_1}\int_0^{V_e} \frac{dV_e}{\cancel{dt}} \cdot dt + \frac{1}{R_1 C}\int_0^{V_e} V_e \, dt = 0$$

$$V_{out\,1} + \frac{R_2}{R_1} V_e + \frac{1}{R_1 C}\int_0^t V_e \, dt + V_{(0)} = 0$$

$$\frac{R_2}{R_1} V_e + \frac{1}{R_1 C}\int_0^t V_e \, dt + V_{(0)} = -V_{out\,1}$$

from the inverting Circuit;

$$V_{out} = -V_{out\,1}$$

$$\Rightarrow V_{out} = \frac{R_2}{R_1} V_e + \frac{1}{R_1 C}\int_0^t V_e \, dt + V_{(0)}$$

multiplying through by R_2/R_2

$$\frac{\cancel{R_2}}{\cancel{R_2}} V_{out} = \frac{\cancel{R_2}}{\cancel{R_2}} \cdot \frac{R_2 V_e}{R_1} + \frac{R_2}{R_2} \cdot \frac{1}{R_1 C}\int_0^t V_e \, dt + V_{(0)}$$

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} * \frac{1}{R_2 C}\int_0^t V_e \, dt + V_{(0)}$$

OR

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e \, dt + V_{(0)}$$

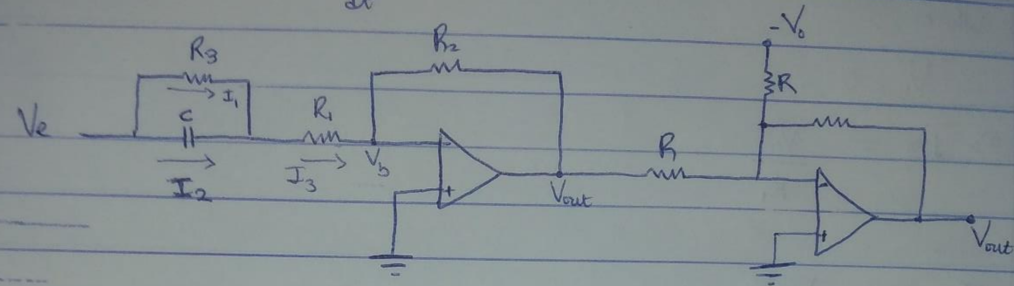
where;

$$G_P = \frac{R_2}{R_1} \longrightarrow \text{Proportional gain}$$

$$G_I = \frac{1}{R_2 C} \longrightarrow \text{Integral gain.}$$

Proportional - Derivative [PD]

$$P(t) = K_p e_p + K_p K_D \frac{d e_p}{dt} + P(0)$$



Effective resistance R is given as;

$$R = \frac{R_1 R_3}{R_1 + R_3} \quad \text{-------} \ast$$

Applying KCL,

$$I_1 + I_2 = I_3$$

$$\Rightarrow I_1 + I_2 - I_3 = 0$$

Also

$$I_4 + I_3 = 0$$

from the circuit;

$$I_1 = \frac{V_e - V_a}{R_3}$$

$$I_2 = \frac{c\,d\,[V_e - V_a]}{dt}$$

$$I_3 = \frac{V_a - V_b}{R_1}$$

where

$$V_b = 0$$

$$\Rightarrow I_3 = \frac{V_a}{R_1}$$

$$I_4 = \frac{V_{out1}}{R_2}$$

Combining with Ohms law

$$\frac{V_e - V_a}{R_3} + c\frac{d}{dt}\left[V_e - V_a\right] - \frac{V_a}{R_1} = 0 \quad \text{-------- (1)}$$

$$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0 \quad \text{-------- (2)}$$

from equation (2)

Making V_a subject of formula.

$$\frac{V_a}{R_1} = -\frac{V_{out1}}{R_2}$$

$$V_a = -\frac{R_1 V_{out1}}{R_2} \quad \text{-------- (3)}$$

Taking Laplace transform of equation 1

$$\frac{V_{e(s)} - V_{a(s)}}{R_3} + SC\left[V_{e(s)} - V_{a(s)}\right] - \frac{V_{a(s)}}{R_1} = 0$$

$$\frac{V_{e(s)}}{R_3} - \frac{V_{a(s)}}{R_3} + SCV_{e(s)} - SCV_{a(s)} - \frac{V_{a(s)}}{R_1} = 0$$

$$\frac{V_{e(s)}}{R_3} + SCV_{e(s)} = \frac{V_{a(s)}}{R_3} + SCV_{a(s)} + \frac{V_{a(s)}}{R_1}$$

$$\left[\frac{1}{R_3} + Sc\right]V_{e(s)} = \left[\frac{1}{R_3} + Sc + \frac{1}{R_1}\right]V_{a(s)} \quad \text{--------- (4)}$$

Substituting eqn 3 into eqn 4

$$\left[\frac{1}{R_3} + Sc\right]V_{e(s)} = \left[\frac{1}{R_3} + Sc + \frac{1}{R_1}\right] * -\frac{R_1 V_{out1}}{R_2}$$

$$\left[\frac{1 + R_3Sc}{\cancel{R_3}}\right]V_{e(s)} = \left[\frac{R_1 + R_1R_3Sc + R_3}{\cancel{R_1}\cancel{R_3}}\right] * -\frac{\cancel{R_1}\, V_{out}}{R_2}$$

$$\left[1 + R_3Sc\right]V_{e(s)} = \frac{-V_{out1}}{R_2}\left[R_1 + R_1R_3Sc + R_3\right]$$

making $-V_{out1(s)}$ subject of formula.

$$\left[R_2 + R_2R_3SC\right]V_{e(s)} = -V_{out1(s)}\left[R_1 + R_1R_3Sc + R_3\right]$$

$$-V_{out1(s)} = \frac{\left[R_2 + R_2R_3SC\right]V_{e(s)}}{\left[R_1 + R_1R_3Sc + R_3\right]}$$

$$-V_{out1}(s) = \frac{V_{e(s)}\left[R_2 + R_2R_3SC\right]/R_1+R_3}{\left[R_1 + R_1R_3SC + R_3\right]/R_1+R_3}$$

$$-V_{out1} = \frac{V_{e(s)}\left[R_2 + R_2R_3SC\right]/R_1+R_3}{\frac{\cancel{R_1+R_3}}{\cancel{R_1+R_3}} + \frac{R_1R_3SC}{R_1+R_3}}$$

$$-V_{out1} = \frac{V_{e(s)}\left[R_2 + R_2R_3Sc\right]/R_1+R_3}{1 + \frac{R_1R_3SC}{R_1+R_3}}$$

from eqn *

$$R = \frac{R_1R_3}{R_1+R_3}$$

$$\Rightarrow -V_{out1} = \frac{V_{e(s)}\left[R_2 + R_2R_3SC\right]/R_1+R_3}{1 + RSC}$$

If $1 \gg RSC$

then

$$-V_{out} = \frac{V_{e(s)}(R_2 + R_2R_3Sc]/R_1+R_3}{1} = \frac{V_{e(s)}(R_2+R_2R_3Sc)}{R_1+R_3}$$

from the inverting circuit.

$$V_{out1} = -V_{out} + V_0$$

$$\Rightarrow V_{out(s)} = \frac{V_{e(s)}(R_2 + R_2 R_3 S C)}{R_1 + R_3} + V_{(0)}$$

$$V_{out(s)} = \frac{R_2 V_{e(s)}}{R_1 + R_3} + \frac{R_2 R_3 S C}{R_1 + R_3} V_{e(s)} + V_{(0)}$$

$$V_{out(s)} = \frac{R_2}{R_1 + R_3} \cdot V_e + \frac{R_2}{R_1 + R_3} \cdot RSC\, V_{e(s)} + V_{(0)}$$

Taking inverse laplace

$$V_{out} = \left[\frac{R_2}{R_1 + R_3}\right] V_e + \left[\frac{R_2}{R_1 + R_2}\right] R_3 C \frac{dV_e}{dt} + V_{(0)}$$

or

$$V_{out} = G_p V_e + G_p G_D \frac{dV_e}{dt} + V_{(0)}$$

where

$$G_p = \left[\frac{R_2}{R_1 + R_3}\right]$$ ----- Proportional gain

$$G_D = \left[R_3 C\right]$$ ---- Derivative gain.