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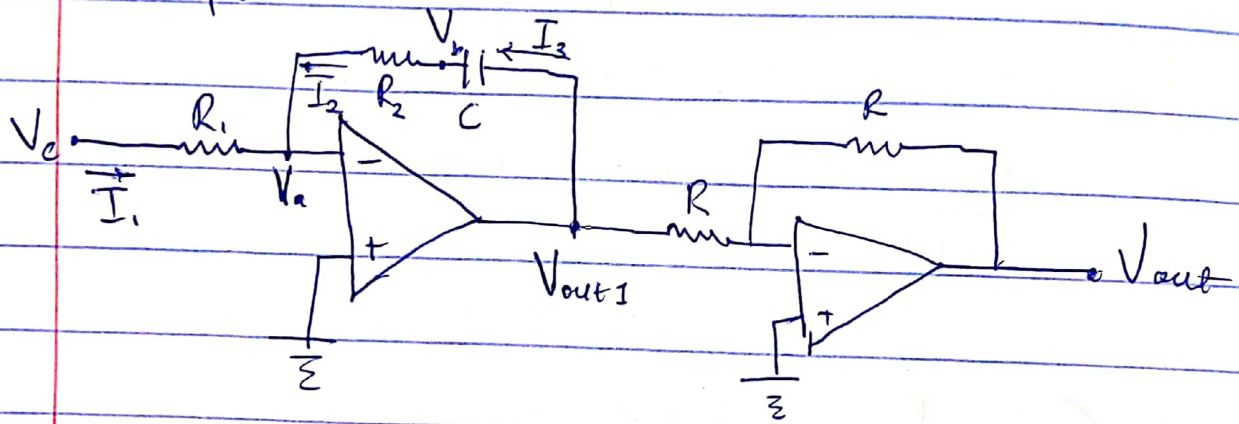
course code/ EEE 561

Proportional Integral Controller using OP-Amps



$$V_a = 0$$

$$I_1 + I_2 = 0, \quad I_3 - I_2 = 0$$

$$I_c = C\frac{dv}{dt}$$

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \quad \text{——} \quad (1)$$

$$C\frac{d[V_{out_1} - V_b]}{dt} - \frac{V_b}{R_2} = 0 \quad \text{——} \quad (2)$$

$$V_b = -\frac{R_2}{R_1} V_e \quad \text{——} \quad (3)$$

Substitute eqn (3) into eqn (2)

$$C\frac{dV_{out_1}}{dt} + \frac{CR_2}{R_1}\frac{dV_e}{dt} + \frac{V_e}{R_1} = 0 \quad — (4)$$

Integrating eqn (4)

$$C\int_0^t \frac{dV_{out_1}}{dt}dt + \frac{CR_2}{R_1}\int_0^t \frac{dV_e}{dt}dt + \frac{1}{R_1}\int_0^t V_e\,dt = 0$$

$$C V_{out_1} + \frac{R_2 C}{R_1} V_e + \cancel{V_e}\frac{1}{R_1}\int_0^t V_e\,dt = 0$$

making V_{out_1} subject of the formular

$$V_{out_1} = -\frac{R_2}{R_1}V_e - \frac{1}{R_1 C}\int_0^t V_e\,dt$$

$$V_{out} = -V_{out\,1} + V(0)$$

$$V_{out} = +\frac{R_2}{R_1}V_e + \frac{1}{R_1 C}\int_0^t V_e\,dt + V(0)$$

$$V_{out} = \frac{R_2}{R_1}V_e + \frac{R_2}{R_1}\frac{1}{R_2 C}\int_0^t V_e\,dt + V(0)$$

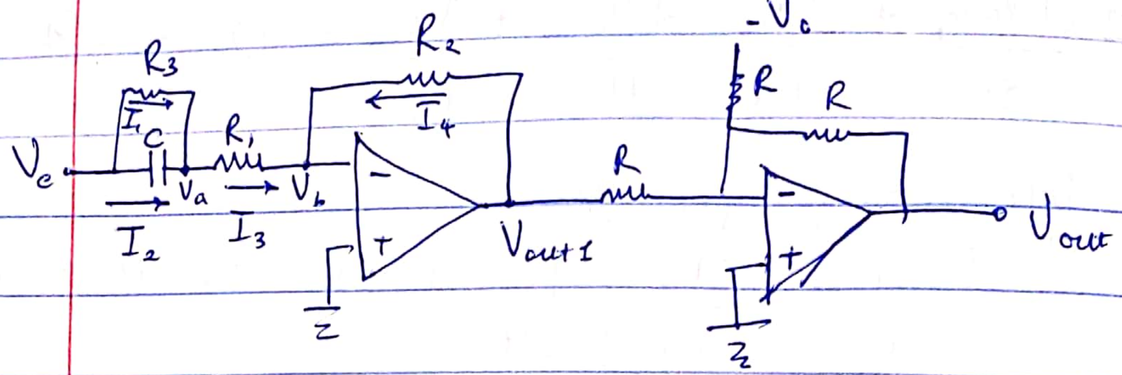
~~Proportional Derivative controller. ckt using OP-Amp~~

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e\,dt + V(0)$$

where $G_P = \frac{R_2}{R_1}$

$$G_I = \frac{1}{R_2 C}$$

Proportional Derivative Controller using OP-Amp



$$I_1 + I_2 - I_3 = 0$$

$$I_4 + I_3 = 0$$

$$\frac{V_e - V_a}{R_3} + C\frac{d[V_e - V_a]}{dt} - \frac{V_a}{R_1} = 0 \quad \text{——— (1)}$$

$$\frac{V_a}{R_1} + \frac{V_{out_1}}{R_2} = 0 \quad \text{——— (2)}$$

substituting eqn (3) into eqn (1)

$$V_a = -\frac{R_1}{R_2} V_{out1} \quad \text{——— (3)}$$

$$\frac{V_e}{R_3} + \frac{R_1}{R_2 R_3} V_{out_1} + C\frac{dV_e}{dt} + \frac{CR_1}{R_2}\frac{dV_{out1}}{dt} + \frac{R_1}{R_1 R_2} V_{out_1} = 0$$

taking laplace transform

$$\frac{V_e(s)}{R_3} + \frac{R_1}{R_2 R_3} V_{out_1}(s) + CsV_e(s) + \frac{sCR_1}{R_2} V_{out_1}(s) + \frac{R_1}{R_1 R_2} V_{out1}(s) = 0$$

$$-V_{out1} = \frac{\frac{V_e(s)}{R_3} + sCV_e(s)}{\frac{R_1}{R_2 R_3} + \frac{sCR_1}{R_2} + \frac{R_1}{R_1 R_2}}$$

$$-V_{out\,1} = \frac{V_e(s)R_2 + sCR_3R_2V_e(s)}{R_1 + sCR_1R_3 + R_3}$$

dovoding through by $R_1 + R_3$

$$-V_{out1} = \frac{\dfrac{V_e(s)R_2 + sCR_3R_2V_e(s)}{R_1+R_3}}{\dfrac{R_1+R_3}{R_1+R_3} + \dfrac{sCR_1R_3}{R_1R_3}}$$

$$= \left(\frac{V_e(s)R_2 + sCR_3R_2V_e(s)}{1 + sCR}\right) \Big/ R_1+R_3$$

$$R = \frac{R_1R_3}{R_1+R_3}$$

taking $sCR << 1$

$$-V_{out1} = \frac{V_e(s)R_2 + sCR_3R_2V_e(s)}{R_1+R_3}$$

$$V_{out_1} = \frac{-V_e(s)R_2 + sCR_3R_2V_e(s)}{-R_1 - R_3}$$

$$V_{out} = -V_{out1} + V(0)$$

$$V_{out} = \frac{V_e(s)R_2 + sCR_3R_2V_e(s)}{R_1+R_3} + V(0)$$

$$V_{out} = \frac{R_2}{R_1+R_3}V_e(s) + \frac{sCR_3R_2}{R_1+R_3}V_e(s) + V_0$$

Taking Inverse laplace transform

$$V_{out} = \frac{R_2}{R_1 + R_3} V_e + \frac{R_2}{R_1 + R_3} R_3 C \frac{dV_e}{dt} + V_0$$

$$V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_0$$

$$G_P = \frac{R_2}{R_1 + R_3}$$

$$G_D = R_3 C$$

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