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16/ENG04/050
Elect/Elect

Proportional Integral Controller Mode

$V_a = 0$

$I_1 + I_2 = 0$ —— (1)

$I_3 - I_2 = 0$ —— (2)

Current through the capacitor

$$I_c = \frac{c\,dV_c}{dt}$$

$$I_1 = \frac{V_e - V_a}{R_1} \quad (V_a = 0)$$

$$= \frac{V_e}{R_1}$$

$$I_2 = \frac{V_b - V_a}{R_2} \quad (V_a = 0)$$

$$= \frac{V_b}{R_2}$$

$$I_3 = C\frac{d}{dt}(V_{out1} - V_b)$$

Sub into eqn 1 and 2

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \quad \text{—— } 1$$

$$C\frac{d}{dt}(V_{out1} - V_b) - \frac{V_b}{R_2} = 0 \quad \text{—— } 2$$

from eqn 1

$$\frac{V_b}{R_2} = -\frac{V_e}{R_1}$$

$$\boxed{V_b = -\frac{R_2}{R_1} V_e}$$

Taking Laplace transform of eqn 2

$$SC(V_{out1}(s) - V_b(s)) - \frac{V_b(s)}{R_2} = 0$$

$$SCV_{out1}(s) = SCV_b(s) + \frac{V_b(s)}{R_2}$$

$$SCV_{out1}(s) = V_b(s)\left(SC + \frac{1}{R_2}\right)$$

Recall $V_b = -\frac{R_2}{R_1} V_e$

$$sC\, V_{out_1}(s) = -\frac{R_2}{R_1} V_e(s) \left( sC + \frac{1}{R_2} \right)$$

$$V_{out_1}(s) = -\frac{R_2}{sCR_1} V_e(s) \left( sC + \frac{1}{R_2} \right)$$

$$V_{out_1}(s) = -\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1} \frac{1}{sCR_2} V_e(s)$$

From the inverting circuit

$$V_{out_1} = -V_{out}$$

$$\therefore V_{out}(s) = -\left( -\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1} \frac{1}{sCR_2} V_e(s) \right)$$

$$V_{out}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1} \frac{1}{sCR_2} V_e(s)$$

Taking inverse Laplace

$$V_{out} = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1} \frac{1}{R_2} \int_0^t V_e(t)\,dt + V_{(0)}$$

$$\left( \text{where } \frac{1}{s} = \int_0^t dt + k \right)$$

$$\boxed{V_{out} = G_P V_e + G_P G_I \int_0^t V_e\,dt + V(0)}$$

where $G_P = \dfrac{R_2}{R_1}$

$G_I = \dfrac{1}{R_2 C}$

Proportional Derivative Controller Mode

$$I_1 + I_2 = I_3 \quad \text{———} \ (1)$$

$$I_3 + I_4 = 0 \quad \text{———} \ (2)$$

$$I_1 = \frac{V_e - V_a}{R_3}$$

$$I_2 = \frac{cd}{dt}(V_e - V_a)$$

$$I_3 = \frac{V_a - V_b}{R_1} \quad (V_b = 0)$$

$$= \frac{V_a}{R_1}$$

$$I_4 = \frac{V_{out1} - V_b}{R_2} \quad (V_b = 0)$$

$$= \frac{V_{out1}}{R_2}$$

$$R = \frac{R_1 R_3}{R_1 + R_3}$$ — effective resistance

Sub into eqn ① and ②

$$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}(V_e - V_a) = \frac{V_a}{R_1} \quad \text{———} \ *$$

$$\frac{V_a}{R_1} + \frac{V_{out_1}}{R_2} = 0 \quad \text{———} \ **$$

from **

$$\frac{V_a}{R_1} = -\frac{V_{out_1}}{R_2}$$

$$\boxed{V_a = -\frac{R_1}{R_2} V_{out_1}}$$

Rearranging *

$$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$$

Taking Laplace transform

$$\frac{V_e(s) - V_a(s)}{R_3} + sC(V_e(s) - V_a(s)) - \frac{V_a(s)}{R_1} = 0$$

(Initial conditions go to zero)

$$\frac{V_e(s)}{R_3} + sC\,V_e(s) = \frac{V_a(s)}{R_1} + \frac{V_a(s)}{R_3} + sC\,V_a(s)$$

$$V_e(s)\left(\frac{1}{R_3} + sC\right) = V_a(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right)$$

Recall $V_a = -\frac{R_1}{R_2} V_{out1}$

$$V_e(s)\left(\frac{1}{R_3} + sC\right) = -\frac{R_1}{R_2} V_{out1}(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right)$$

$$V_e(s)\left(\frac{1 + R_3 sC}{\cancel{R_3}}\right) = -\frac{\cancel{R_1}}{R_2} V_{out1}(s)\left(\frac{R_3 + R_1 + sC(R_1 R_3)}{\cancel{R_1}\cancel{R_3}}\right)$$

$$V_e(s)(1 + sC(R_3)) = -\frac{V_{out1}(s)}{R_2}(R_3 + R_1 + sC(R_1 R_3))$$

$$-V_{out1}(s) = \frac{V_e(s)(1 + sC(R_3))R_2}{(R_1 + R_3 + sC(R_1 R_3))}$$

$$-V_{out1}(s) = \frac{V_e(s)(R_2 + sC(R_2 R_3))}{(R_1 + R_3 + sC(R_1 R_3))}$$

Dividing numerator & denominator by $R_1+R_3$

$$-V_{out1}(s) = \frac{V_e(s)(R_2+s(R_2R_3))/R_1+R_3}{\frac{R_1+R_3}{R_1+R_3} + \frac{sCR_1R_3}{R_1+R_3}}$$

Recall $R = \frac{R_1R_3}{R_1+R_3}$

$$\cancel{-V_{out1}(s) = \frac{V_e(s)(R_2+s(R_2R_3)}{R_1+R}}$$

$$-V_{out1}(s) = \frac{V_e(s)(R_2+s(R_2R_3))/R_1+R_3}{1+sCR}$$

If $sCR << 1$

$$-V_{out1}(s) = \frac{V_e(s)(R_2+s(R_2R_3))}{R_1+R_3}$$

from the inverting circuit

$V_{out1} = -V_{out} + V_0$

$$\therefore -(-V_{out}(s)+V(0)) = \frac{V_e(R_2+s(R_2R_3))}{R_1+R_3}$$

$$V_{out}(s) - V(0) = \frac{V_e(s)R_2}{R_1+R_3} + \frac{s(R_2R_3\,V_e(s))}{R_1+R_3}$$

$$V_{out}(s) = \frac{R_2}{R_1 + R_3} V_e(s) + \frac{R_2}{R_1 + R_3} R_3 C (s V_e(s) + V(0))$$

Taking inverse Laplace

$$V_{out} = \frac{R_2}{R_1 + R_3} V_e + \frac{R_2}{R_1 + R_3} R_3 C \frac{dV_e}{dt} + V_0$$

$$\boxed{V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_0}$$

where $G_P = \frac{R_2}{R_1 + R_3}$

$G_D = \cancel{R_3 C}$