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PI controller

$V_a = 0$

$I_1 + I_2 = 0 \quad \text{------} \quad 1$

$I_3 - I_2 = 0 \quad \text{------} \quad 2$

current flowing through the capacitor

$$I_c = C\frac{dv}{dt}$$

$$I_1 = \frac{V_e - V_a}{R_1} \quad \text{where } V_a = 0$$

$$= \frac{V_e}{R_1}$$

$$I_2 = \frac{V_b - V_a}{R_2} \quad (V_a = 0)$$

$$= \frac{V_b}{R_2}$$

$$I_3 = \frac{Cd}{dt}(V_{out1} - V_b)$$

sub into eqn (1) and (2)

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \quad \text{--------} \quad (1)$$

$$\frac{Cd}{dt}(V_{out1} - V_b) - \frac{V_b}{R_2} = 0 \quad \text{------} \quad (2)$$

from eqn (1)

$$\frac{V_b}{R_2} = -\frac{V_e}{R_1}$$

making V_b the subject of the formular

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$v_b = -\frac{R_2}{R_1} v_e$

Taking the laplace transform of eqn (2)

~~Sc (Vout1~~

$sC(V_{out1}(s) - V_b(s)) - \frac{V_b(s)}{R_2} = 0$

$sC\,V_{out1}(s) = sC\,V_b(s) + \frac{V_b(s)}{R_2}$

$sC\,V_{out1}(s) = V_b(s)\left(sC + \frac{1}{R_2}\right)$

recall, $v_b = -\frac{R_2}{R_1} v_e$

$sC\,V_{out1}(s) = -\frac{R_2}{R_1} V_e(s)\left[sC + \frac{1}{R_2}\right]$

$V_{out1}(s) = -\frac{R_2}{sCR_1} V_e(s)\left(sC + \frac{1}{R_2}\right)$

$V_{out}(s) = -\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{sCR_2} V_e(s)$

from the inverting circuit

$V_{out1} = -V_{out}$

Therefore

$V_{out}(s) = -\left(-\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{sCR_2} V_e(s)\right)$

$V_{out}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{sCR_2} V_e(s)$

~~Vout =~~ Taking the inverse laplace

$V_{out} = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1 R_2}\left(\int_0^t V_e(t)\,dt + V_{C0}\right)$

where $1/s = \int_0^t dt + k$

$$V_{out} = G_p V_e + G_p G_I \int_0^t V_e\,dt + V_c(0)$$

where $G_p = \frac{R_2}{R_1}$

$G_I = \frac{1}{R_2 C}$

2 proportional Derivative ~~ctr~~ controller mode

$I_1 + I_2 = I_3$ ------ (1)

$I_3 + I_4 = 0$ --- ------- (2)

$I_1 = \frac{V_e - V_a}{R_3}$

$I_2 = \frac{C\,d}{dt}(V_e - V_a)$

$I_3 = \frac{V_a - V_b}{R_4}$ ($V_b = 0$)

$= \frac{V_a}{R_4}$

$I_4 = \frac{V_{out1} - V_b}{R_2}$ ($V_b = 0$)

$= \cancel{V_{out1}}\ \frac{V_{out1}}{R_2}$

$R = \frac{R_1 R_3}{R_1 + R_3}$ -------... effective resistance

Subs int eqn (1) and eqn (2)

$\frac{V_e - V_a}{R_3} + \frac{C\,d}{dt}(V_e - V_a) = \frac{V_a}{R_1}$ ----- 1

$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0$ ------ ---- 2

From eqn 1 and 2

$\frac{V_a}{R_1} = \frac{-V_{out1}}{R_2}$

$$v_a = -\frac{R_1}{R_2} v_{out1}$$

from eqn 1

$$\frac{v_e - v_a}{R_3} + C\frac{d}{dt}(v_e - v_a) - \frac{v_a}{R_1} = 0$$

taking Laplace transform

$$\frac{v_e(s) - v_a(s)}{R_3} + sC(v_e(s) - v_a(s)) - \frac{v_a(s)}{R_1} = 0$$

(initial conditions go to zero)

$$\frac{v_e(s)}{R_3} + sCv_e(s) = \frac{v_a(s)}{R_1} + \frac{v_a(s)}{R_3} + sC\,v_a(s)$$

$$v_e(s)\left(\frac{1}{R_3} + sC\right) = v_a(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right)$$

recall $v_a = -\frac{R_1}{R_2} v_{out1}$

$$v_e(s)\left(\frac{1}{R_3} + sC\right) = -\frac{R_1}{R_2} v_{out1}(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right)$$

taking lcm

$$v_e(s)\left(\frac{1 + R_3 sC}{\cancel{R_3}}\right) = -\frac{\cancel{R_1}}{R_2} v_{out1}(s)\left(\frac{R_3 + R_1 + sCR_1R_3}{\cancel{R_1}\cancel{R_3}}\right)$$

$$v_e(s)\,(1 + sCR_3) = -\frac{v_{out1}(s)}{R_2}(R_3 + R_1 + sCR_1R_3)$$

$$-v_{out1}(s) = \frac{v_e(s)\,(1 + sCR_3)R_2}{(R_1 + R_3 + sCR_1R_3)}$$

$$-v_{out1}(s) = \frac{v_e(s)(R_2 + sCR_2R_3)}{(R_1 + R_3 + sCR_1R_3)}$$

dividing numerator and denominator by $R_1 + R_3$

$$-v_{out}(s) = \frac{v_e(s)\,(R_2 + sCR_2R_3)/R_1 + R_3}{\frac{R_1 + R_3}{R_1 + R_3} + \frac{sCR_1R_3}{R_1 + R_3}}$$

recall $R = \frac{R_1 R_3}{R_1 + R_3}$

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$$-v_{out1}(s) = \frac{V_e(s)(R_2 + sC R_2 R_3)/R_1 + R_3}{1 + sCR}$$

If $sCR << 1$

$$-v_{out1}(s) = \frac{V_e(s)(R_2 + sCR_2 R_3)}{R_1 + R_3}$$

from the inverting circuit

$v_{out1} = -v_{out} + v_0$

then the $(-v_{out}(s) + v(0)) = V_e \frac{(R_2 + sCR_2R_3)}{R_1 + R_3}$

$$v_{out}(s) - v(0) = \frac{v_e R_2}{R_1 + R_3} + \frac{sCR_2 R_3 v_e(s)}{R_1 + R_3}$$

$$v_{out} = \frac{R_2}{R_1 + R_3} v_e(s) + \frac{R_2}{R_1 + R_3} R_3 Cs V_e(s) + v(0)$$

Taking inverse laplace

$$v_{out} = \frac{R_2}{R_1 + R_3} v_e + \frac{R_2}{R_1 + R_3} R_3 C \frac{dv_e}{dt} + v_0$$

$$v_{out} = G_P V_e + G_P G_D \frac{dv_e}{dt} + v_0$$

where $G_P = \frac{R_2}{R_1 + R_3}$

$G_D = R_3 C$