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17/ENG04/023
Elect/Elect Engineering, 400lvl

EEE 441

2.) ROUTH'S STABILITY CRITERION

Consider a closed-loop function

$$H(s) = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n} = \frac{B(s)}{A(s)} \quad \dots (i)$$

Where the a_i's and b_i's are real constants and $m \leq n$. An alternative to factoring the denominator polynomial, Routh's stability criterion, determines the number of closed-loop poles in the right-half s plane.

The Algorithm described below, like the stability criterion requires the order A(s) to be finite:

~~2)a When entire rows is zero on the Routh Hurwi~~

1.) Factor out any roots at the origin to obtain the polynomial, and multiply by -1 if necessary to obtain

$$a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n = 0 \quad \dots (ii)$$

where $a_0 \neq 0$ and $a_n > 0$.

2.) If the order of the resulting polynomial is at least two and any coefficient a_i is zero or negative, the polynomial has at least one root with a nonnegative real part. To obtain the precise number of roots with nonnegative real part, proceed as follows. Arranged the coefficients of the polynomial, and values subsequently calculated from them as shown below

$$\begin{array}{lllll}
s^n & a_0 & a_2 & a_4 & a_6 \; \dots \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \; \dots \\
s^{n-2} & b_1 & b_2 & b_3 & b_4 \; \dots \\
s^{n-3} & c_1 & c_2 & c_3 & c_4 \; \dots \\
s^{n-4} & d_1 & d_2 & d_3 & d_4 \\
\vdots & \vdots & \vdots & & \\
s^2 & e_1 & e_2 & & \\
s^1 & f_1 & & & \\
s^0 & g_0 & & &
\end{array} \quad \text{--- (iii)}$$

where the coefficients b_i are

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \quad \text{--- (iv)}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \quad \text{--- (v)}$$

$$b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1} \quad \text{--- (vi)}$$

$\vdots$

generated until all subsequent coefficients are zero. Similarly, cross multiply the coefficients of the two previous rows to obtain the c_i and d_i etc.

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \quad \text{--- (vii)}$$

$$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1} \quad \text{--- (viii)}$$

$$c_3 = \frac{b_1 a_7 - a_1 b_4}{b_1} \quad \text{--- (ix)}$$

$\vdots$

$$d_1 = \frac{c_1 b_2 - b_1 c_2}{c_1} \quad \text{--- (x)}$$

$$d_2 = \frac{c_1 b_3 - b_1 c_3}{c_1} \quad \text{--- (xi)}$$

until the nth row of the array has been completed. Missing coefficients are replaced by zeros. The resulting array is called the Routh array. The powers of s are not considered to be part of the array. We can think of them as labels. The column beginning with a_0 is considered to be the first column of the array.

The Routh array is seen to be triangular. It can be shown that multiplying a row by a positive number to simplify the calculation of the next row does not affect the outcome of the application of the Routh criterion.

3.) Count the number of sign changes in the first column of the array. It can be shown that a necessary and sufficient condition for all roots of eqn(ii) to be located in the left-hand plane is that all a_i are positive and all of the ~~coeffici~~ coefficients in the first column be positive

2a) When entire rows is zero on the Routh table

If all the coefficients in a row are zero, a pair of roots of equal magnitude and opposite sign is indicated. These could be two real roots with equal magnitudes and opposite signs or two conjugate imaginary roots. The zero row is replaced by taking the coefficients ~~of~~ of $dP(s)/ds$, where $P(s)$, called the ~~auxil~~ auxiliary polynomial, is obtained from the values in the row above the zero row. The pair of roots can be found by solving $dP(s)/ds = 0$

Note that the auxiliary polynomial ~~ato~~ always has even degree. It can be shown that an auxiliary ~~plo~~ polynomial of degree $2n$ has n pairs of equal magnitude and opposite sign.

Example: Use of Auxiliary Polynomial

Consider the quintic equation $A(s) = 0$ where $A(s)$ is

$$s^5 + 2s^4 + 24s^3 + 48s^2 - 50$$

The Routh Array starts off as

$$\begin{array}{llll} s^5 & 1 & 24 & -25 \\ s^4 & 2 & 48 & -50 \\ s^3 & 0 & 0 & \end{array}$$

← auxiliary polynomial $P(s)$

The auxiliary polynomial $P(s)$ is

$$P(s) = 2s^4 + 48s^2 - 50$$

Which indicates that $A(s) = 0$ must have two pairs of roots of equal magnitude and opposite sign, which are also roots of the auxiliary polynomial equation $P(s) = 0$. Taking the derivative of $P(s)$ with respect to S we obtain

$$\frac{dP(s)}{ds} = 8s^3 + 96s$$

So the s^3 row is as shown below and the Routh array is

$$\begin{array}{llll} s^5 & 1 & 24 & -25 \\ s^4 & 2 & 48 & 50 \\ s^3 & 8 & 96 & \\ s^2 & 24 & -50 & \\ s^1 & 112.7 & 0 & \\ s^0 & -50 & & \end{array}$$

← coefficients of $dP(s)/ds$

There is a single change of sign in the first column of the resulting array, indicating that there $A(s) = 0$ has one root with the positive real part. Solving the auxiliary polynomial equation,

$$2s^4 + 48s^2 - 50 = 0$$

yields the remaining roots, namely, from

$$s^2 = 1, \quad s^2 = -25,$$

$$s = \pm 1, \quad s = \pm j5$$

So the original equation can be factored as

$$(s+1)(s-1)(s+j5)(s-j5)(s+2) = 0$$

2b.) To determine the poles on the $j\omega$ axis

~~As the entries from s^5~~

The rows of zeros indicates the possibility of $j\omega$ roots.

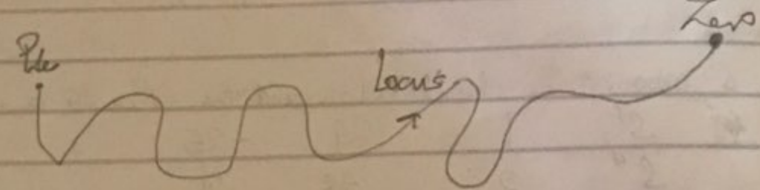
As the entries from s^4 to s^1 are looking at the even polynomials and there are no sign changes, so all 4 poles must exist on the $j\omega$ axis

1. Root Locus Technique

Root means root of characteristic equation (poles of the closed loop)

Locus means path

ROOT LOCUS means closed ~~pat~~ loop poles path by varying the gain K from zero to infinity



- Controller Design Using Root Locus

Letting the CL characteristic equation (CLCE) be the polynomial equation, one can use the Root Locus technique to find how a positive controller design parameter affects the resulting CL poles from which one can choose a right value for the controller parameter

The Root Locus Method

- No matter what we pick 'K' to be, the closed-loop system must always have 'n' poles, where n is the number of poles of G(s)

- The Root locus must have 'n' branches, each branch starts at a pole of G(s) and goes to a zero of G(s)

- If $G(s)$ has more poles than zeros (as is often the case), $m<n$ and we say that $G(s)$ has zeros at infinity. In this case, the limit of $G(s)$ as $s \to \infty$ is zero

- The number of zeros at infinity is $n-m$, the number of poles minus the number of zeros, and is the number of branches of the root locus that go to infinity (asymptotes)

- Since the root locus is actually the locations of all possible closed loop poles, from the root locus, we can select a gain such that our closed-loop system will perform the way we want. If any of the selected poles are on the right-half plane, the closed-loop system will be unstable. The poles that are closest to the imaginary axis have the greatest influence on the closed-loop response, so even though the system has three or four poles, it may still act like a second or even first order system depending on the location(s) of the dominant pole(s).

~~Methods of Obtaining Root Locus~~

Root locus is the method of graphically displaying the roots of a polynomial equation having the following form on the complex plane when the parameter K varies from 0 to ∞

$$1+K\cdot G(s)=0 \quad \text{or} \quad 1+K\cdot\frac{N(s)}{D(s)}=0$$

Steps to Sketch Root Locus

1.) Transform the closed-loop characteristics equation into the standard form for sketching root locus:

$$1+K\cdot\frac{N(s)}{D(s)}=0$$

2.) Find the open-loop zeros, z_i, and open-loop poles, P_i.
Mark the open loop poles and zeros on the complex plane.
Use (') prime to represent open loop poles and (;) to represent open-loop zeros

3.) Determine the real axis segments that are on the root locus by applying Rule 4

4.) Determine the number of asymptotes and the corresponding intersection S_0 and angles ϕ_k by applying Rules 2 and 5

Rule 2: Root locus starts at open-loop poles (when $K=0$) and ends at open-loop zeros (when $K=\infty$).

Rule 5: If number of poles NP exceeds the number of zeros NZ, then as $K \to \infty$, (NP − NZ) branches will become as asymptotic to straight lines. These straight lines intersect the real axis with angle ϕ_K and S^0

5.) (If necessary) Determine the break-away and break-in points using Rule 6: Breakaway and/or break-in (arrival) points should be the solutions to this equation: $\frac{d}{ds}\left(\frac{N(s)}{D(s)}\right)=0$ or $\frac{d}{ds}\left(\frac{D(s)}{N(s)}\right)=0$

6.) (If necessary) Determine the departure and arrival angles using Rule 7

Rule 7: The departure angle for a pole P_i (the arrival angle for a zero Z_i) can be calculated by slightly modifying the following equation:

$$\angle(s-z_1)+\angle(s-z_2)+\boxed{\ldots}+\angle(s-z_{N_z})-\angle(s-p_1)-\angle(s-p_2)-\boxed{\ldots}-\angle(s-p_{N_p})=180^\circ$$

7.) (If necessary) Determine the imaginary axis crossings using Rule 8:

If the root locus passes through the imaginary axis (the stability boundary), the crossing point $j\omega$ and the corresponding gain K can be found as follows:

- Replace s in the left side of the closed-loop characteristic equation with $j\omega$ to obtain the real and imaginary parts of the resulting complex number

- ~~Replace~~ Set the real and imaginary parts to zero, and solve for ω and K. This will tell you what values of K and what points on the $j\omega$ axis the roots will cross

8.) Use the information from steps 1-7 and rules 1-3 to sketch ~~out~~ the root locus

Rule 1: The number of branches of the root locus is equal to the number of closed-loop poles.

Rule 2: Root locus starts at open-loop poles (when $K=0$) and ends at open-loop zeros (when $K=\infty$)

Rule 3: Root locus is symmetric about the real axis