SULEIMAN ABDULLAHI EZHIN
17/ENG05/046
MECHATRONICS ENGINEERING.

Derive the analysis for the output voltage in using operational amplifier for
i) Proportional Integral Controller mode
ii) Proportional Derivative Controller mode.

Solution

For proportional Integral Controller mode.

$$I_1 + I_2 = 0 \quad \text{——— (1)}$$
$$I_3 - I_2 = 0 \quad \text{——— (2)}$$

The Current through the Capacitor is
$$I_c = C\frac{dV_c}{dt}$$

Note: $I_1 = \frac{V_c - V_a}{R_1}$ But $(V_a = 0)$

$\therefore I_1 = \frac{V_c}{R_1}$

$I_2 = \frac{V_b - V_a}{R_2}$ $(V_a = 0)$ ——— (*)

$\therefore I_2 = \frac{V_b}{R_2}$

$I_3 = \frac{Cd}{dt}(V_{out}1 - V_b)$

Substitute eqn (*) Into eqn (1) & (2)

$$\frac{V_c}{R_1} + \frac{V_b}{R_2} = 0 \quad \text{——— (3)}$$

$$\frac{Cd}{dt}(V_{out}1 - V_b) - \frac{V_b}{R_2} = 0 \quad \text{——— (4)}$$

eqn (3) becomes

$$V_b = -\frac{R_2}{R_1} V_c$$

The laplace Transform of eqn (4) becomes

$$SC\left(V_{out1}(s) - V_b(s)\right) - \frac{V_b(s)}{R_2} = 0$$

$$SC\,V_{out1}(s) = SC\,V_b(s) + \frac{V_b(s)}{R_2}$$

$$SC\,V_{out1}(s) = V_b(s)\left(SC + \frac{1}{R_2}\right)$$

But $V_b = -\frac{R_2}{R_1} V_c$

$$SC\,V_{out1}(s) = -\frac{R_2}{R_1} V_c(s)\left(SC + \frac{1}{R_2}\right)$$

$$V_{out1}(s) = \frac{-R_2}{SCR_1} V_c(s)\left(SC + \frac{1}{R_2}\right)$$

$$V_{out1}(s) = \frac{-R_2}{R_1} V_c(s) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_c(s)$$

from the Inverting Circuit

$$V_{out1} = -V_{out}$$

$$\therefore -V_{out}(s) = -\left(-\frac{R_2}{R_1} V_c(s) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_c(s)\right)$$

$$V_{out}(s) = \frac{R_2}{R_1} V_c(s) + \frac{R_2}{R_1}\frac{1}{SCR_2} V_c(s)$$

taking the Inverse laplace transform

$$V_{out} = \frac{R_2}{R_1} V_c(s) + \frac{R_2}{R_1}\frac{1}{R_2}\int_0^t V_c(t)\,dt + V(0)$$

$\left(\text{Where } \frac{1}{S} = \int_0^t dt + k\right)$ But $G_P = \frac{R_2}{R_1}$ & $G_I = \frac{1}{R_2 C}$

$$\boxed{V_{out} = G_P V_c + G_P G_I \int_0^t V_c\,dt + V(0)}$$

For PD Controller

$$I_1 + I_2 = I_3 \quad \text{--- (1)}$$

$$I_3 + I_4 = 0 \quad \text{--- (2)}$$

$$I_1 = \frac{V_c - V_a}{R_3}$$

$$I_2 = \frac{cd}{dt}(V_c - V_a)$$

$$I_3 = \frac{V_a - V_b}{R_1} \quad (V_b = 0)$$

$$= \frac{V_a}{R_1}$$

$$I_4 = \frac{V_{out}1 - V_b}{R_2} \quad (V_b = 0)$$

$$= \frac{V_{out}1}{R_2}$$

--- (*)

$$R = \frac{R_1 R_3}{R_1 + R_3} \leftarrow \text{effective resistance}$$

Substituting eqn (*) into eqn (1) & (2)

$$\frac{V_c - V_a}{R_3} + \frac{cd}{dt}(V_c - V_a) = \frac{V_a}{R_1} \quad \text{--- (3)}$$

$$\frac{V_a}{R_1} + \frac{V_{out}1}{R_2} = 0 \quad \text{--- (4)}$$

eqn (4) becomes

$$\frac{V_a}{R_1} = \frac{-V_{out}1}{R_2}$$

$$V_a = -\frac{R_1}{R_2} V_{out}1$$

from eqn (3) re-arranging

$$\frac{V_c - V_a}{R_3} + \frac{cd}{dt}(V_c - V_a) - \frac{V_a}{R_1} = 0$$

The laplace transform becomes

$$\frac{V_c(s) - V_a(s)}{R_2} + sc\left(V_c(s) - V_a(s)\right) - \frac{V_a(s)}{R_1} = 0$$

Initial Conductors becomes zero

$$\frac{V_c(s)}{R_3} + scV_c(s) = \frac{V_a(s)}{R_1} + \frac{V_a(s)}{R_3} + scV_a(s)$$

$$V_c(s)\left(\frac{1}{R_3} + sc\right) = V_a(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sc\right)$$

recall, $V_a = -\frac{R_1}{R_2} V_{out\,1}$

$$V_c(s)\left(\frac{1}{R_3} + sc\right) = \frac{-R_1}{R_2} V_{out\,1}(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + sc\right)$$

$$V_c(s)\left(\frac{1 + R_3 sc}{\cancel{R_3}}\right) = \frac{-\cancel{R_1}}{R_2} \text{Voltage } 1(s)\left(\frac{R_3 + R_1 + scR_1R_3}{\cancel{R_1}\cancel{R_3}}\right)$$

$$V_c(s)(1 + scR_3) = -\frac{V_{out\,1}(s)}{R_2}\left(R_3 + R_1 + scR_1R_3\right)$$

$$-V_{out\,1}(s) = \frac{V_c(s)(1 + scR_3)R_2}{(R_1 + R_3 + scR_1R_3)}$$

$$-V_{out\,1}(s) = \frac{V_c(s)(R_2 + scR_2R_3)}{(R_1 + R_3 + scR_1R_3)}$$

Dividing the Numerator & denominator by $R_1 + R_3$

$$-V_{out\,1}(s) = \frac{V_c(s)\left(R_2 + s(R_2R_3)\right)/R_1 + R_3}{\frac{R_1 + R_3}{R_1 + R_3} + \frac{scR_1R_3}{R_1 + R_3}}$$

recall, $R = \frac{R_1R_3}{R_1 + R_3}$

$$-V_{out\,1}(s) = \frac{V_c(s)\left(R_2 + scR_2R_3\right)/R_1 + R_3}{1 + scR}$$

If $scR \ll 1$

$$-V_{out\,1}(s) = \frac{V_c(s)\left(R_2 + scR_2R_3\right)}{R_1 + R_3}$$

from the Inverting Circuit

$$V_{out\,1} = -V_{out} + V_o$$

Scanned with CamScanner

$$\therefore \ \left(-V_{out}(s) + V(0)\right) = \frac{V_c\left(R_2 + SC R_2 R_3\right)}{R_1 + R_3}$$

$$V_{out}(s) - V(0) = \frac{V_c(s) R_2}{R_1 + R_3} + \frac{SC R_2 R_3}{R_1 + R_3} V_c(s)$$

$$V_{out}(s) = \frac{R_2}{R_1 + R_3} V_c(s) + \frac{R_2}{R_1 + R_3} R_3 C S V_c(s) + V(0)$$

taking the laplace transform

$$V_{out} = \frac{R_2}{R_1 + R_3} V_c + \frac{R_2}{R_1 + R_3} R_3 C \frac{dV_c}{dt} + V_0$$

But $G_p = \frac{R_2}{R_1 + R_3}$ & $G_D = R_3 C$

$$\boxed{\therefore \ V_{out} = G_p V_c + G_p G_D \frac{dV_c}{dt} + V_0}$$