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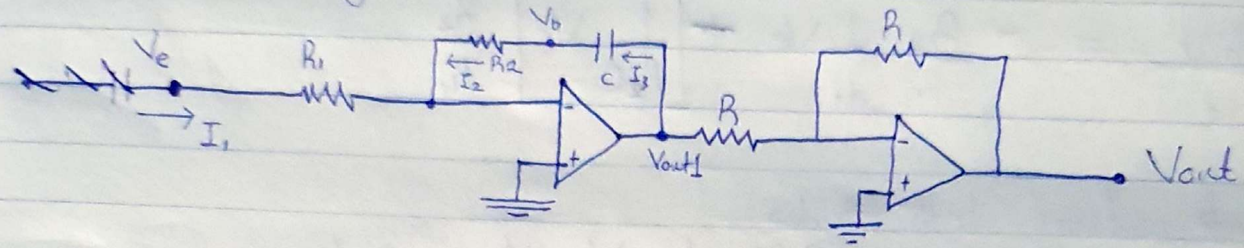
Mechatronics Engr
MCT 511 - Process Automation

Assignment.

1) Proportional Integral Controller Mode



$I_1 + I_2 = 0$

$I_3 - I_2 = 0$

$I_c = C \frac{dV_c}{dt}$

Combining this with ohm's law

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \quad \text{——— (i)}$$

$$C\frac{d}{dt}\left[V_{out_1} - V_b\right] - \frac{V_b}{R_2} = 0 \quad \text{——— (ii)}$$

From (i)

$$V_b = -\frac{R_2}{R_1} V_e \quad \text{——— (iii)}$$

Substituting (iii) into (ii)

$$C\frac{dV_{out_1}}{dt} - C\frac{d}{dt}\left[-\frac{R_2}{R_1} V_e\right] - \frac{1}{R_2}\left[-\frac{R_2}{R_1} V_e\right] = 0$$

$$C\frac{dV_{out}}{dt} + C\frac{R_2}{R_1}\frac{d}{dt}V_e + \frac{R_2}{R_1}\left[\frac{1}{R_2 C}V_e\right] = 0$$

Multiplying allthrough by $1/c$

$$\frac{dV_{out}}{dt} + \frac{R_2}{R_1}\frac{d}{dt}V_e + \frac{R_2}{R_1}\left[\frac{1}{R_2 C}V_e\right] = 0$$

$$\frac{dV_{out}}{dt} = -\frac{R_2}{R_1}\frac{d}{dt}V_e + \left[\frac{1}{R_2 C}V_e\right]\frac{R_2}{R_1}$$

Integrating both sides

$$V_{out} = -\frac{R_2}{R_1}V_e - \frac{R_2}{R_1}\int_0^t \frac{1}{R_2 C}V_e\,dt + V_{(0)}$$

After Inverting

$$V_{out} = \frac{R_2}{R_1}V_e + \frac{R_2 1}{R_1 R_2 C}\int_0^t V_e\,dt + V_{(0)}$$

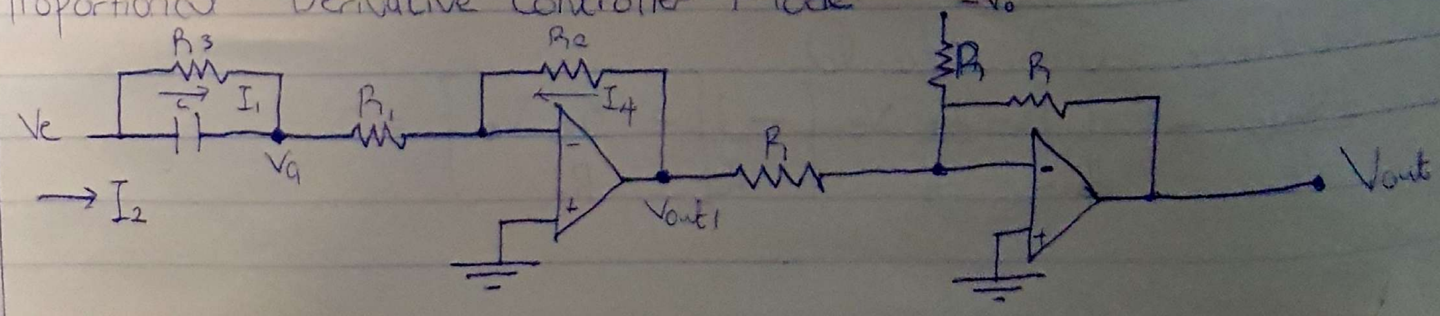
$$\therefore V_{out} = G_P V_e + G_P G_I \int_0^t V_e\,dt + V_{(0)}$$

Where

$G_P = \frac{R_2}{R_1}$ Proportional Gain

$G_I = 1/R_2C$ Integral Gain

2 Proportional Derivative Controller Mode



By KCL

$I_1 + I_2 - I_3 = 0$

$I_4 + I_3 = 0$

Combining with ohm's Law

$$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}[V_e - V_a] - \frac{V_a}{R_1} = 0 \quad \text{———} \quad (a)$$

$$\frac{V_{out}1}{R_2} + \frac{V_a}{R_1} = 0 \quad \text{———} \quad (b)$$

~~Fo~~ From eqn (b)

$$V_a = -\frac{R_1}{R_2} V_{out} \quad \text{———} \quad (c)$$

Substituting (c) into (a)

$$\frac{V_e}{R_3} - \frac{1}{R_3}\left[-\frac{R_1}{R_2}V_{out}\right] + C\frac{dV_e}{dt} - C\frac{d}{dt}\left[-\frac{R_1}{R_2}V_{out}\right] - \left[-\frac{R_1}{R_2}V_{out}\right]\frac{1}{R_1} = 0$$

$$\frac{V_e}{R_3} + \frac{R_1}{R_2R_3}V_{out} + C\frac{dV_e}{dt} + \frac{R_1}{R_2}C\frac{d}{dt}V_{out} + \frac{1}{R_2}V_{out} = 0$$

Multiplying althrough by R_3

$$V_e + \frac{R_1}{R_2}V_{out} + R_3C\frac{dV_e}{dt} + \frac{R_1R_3}{R_2}C\frac{dV_{out}}{dt} + \frac{R_3}{R_2}V_{out} = 0$$

$$\frac{R_1}{R_2}V_{out} + \frac{R_1R_3}{R_2}C\frac{dV_{out}}{dt} + \frac{R_3}{R_2}V_{out} = -V_e - R_3C\frac{dV_e}{dt}$$

$$\left[\frac{R_1 + R_3}{R_2}\right]V_{out} + \frac{R_1R_3}{R_2}C\frac{dV_{out}}{dt} = -V_e - R_3C\frac{dV_e}{dt}$$

Multiply allthrough by $\left[\frac{R_2}{R_1+R_3}\right]$

$$V_{out} + \left[\frac{R_1}{R_1+R_3}\right] R_3 C \frac{dV_{out}}{dt} = -\left[\frac{R_2}{R_1+R_3}\right] V_e - \left[\frac{R_2}{R_1+R_3}\right] R_3 C \frac{dV_e}{dt}$$

After Inverting

$$V_{out} + \left[\frac{R_1}{R_1+R_3}\right] R_3 C \frac{dV_{out}}{dt} = \left[\frac{R_2}{R_1+R_3}\right] + \left[\frac{R_2}{R_1+R_3}\right] R_3 C \frac{dV_e}{dt}$$

$$\therefore V_{out} = \left[\frac{R_2}{R_1+R_3}\right] V_e + \left[\frac{R_2}{R_1+R_3}\right] R_3 C \frac{dV_e}{dt} + V_{(0)}$$

OR

$$V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_{(0)}$$

where

$G_P = \frac{R_2}{R_1+R_3}$ — Proportional Gain

$G_D = R_3 C$ — Derivative Gain.