I am Offering both MCT 511 and EEE 561

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MCT 511 Assignment / EEE561
Process Automation /

Solution

1) Proportional Integral Controller Mode.
2) Proportional Derivative Controller Mode.

For Proportional Integral Controller;

$$C\frac{d}{dt}\left[V_{out} - V_b\right] - \frac{V_b}{R_2} = 0$$

For Proportional Derivative Controller;

$$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}\left[V_e - V_a\right] - \frac{V_a}{R_1} = 0$$

$$\frac{V_{out}1}{R_2} + \frac{V_a}{R_1} = 0$$

for PI; $C\frac{d}{dt}\left[V_{out} - V_b\right] - \frac{V_b}{R_2} = 0$ — — — — equ (i)

$$C\frac{d}{dt}\left[V_{out} - V_b\right] = \frac{V_b}{R_2}$$

$$\frac{d}{dt}\left[V_{out} - V_b\right] = \frac{V_b}{R_2 C}$$

to find V_b;

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0$$

$$\frac{V_b}{R_2} = \frac{V_e}{R_1}$$

$\therefore \quad V_b = \frac{R_2}{R_1} V_e$ — — — — — eqn(ii)

$$\cancel{d}V_{out} - V_b = \int \frac{V_b}{R_2 C} dt$$

$$V_{out} = \int_0^t \frac{V_b}{R_2 C} dt + V_b$$

$$= \cancel{\int_0^t \frac{\frac{R_2}{R_1} V_e}{R_2 C} dt} + \cancel{\left[\frac{R_2 V_e}{R_1}\right]}$$

$$= \cancel{\frac{\frac{R_2}{R_1}}{R_2 C} V_e dt}$$

$$= \frac{-1}{R_2 C} \int_0^t -\frac{R_2}{R_1} V_e dt + \left[-\frac{R_2}{R_1} V_e\right]$$

$$= -\frac{1}{R_2 C} \frac{R}{R_1} \int_0^t V_e dt - \frac{R_2}{R_1} V_e$$

$$= -\left[\frac{R_2}{R_1} V_e + \frac{R_e}{R_1} \frac{1}{R_2 C} \int_0^t V_e dt\right] + V_{(0)}$$

then Inversion takes place $\sim$

that is; $V_{out} = \cancel{\frac{R_2}{R_1} V_e + \frac{R_e}{R_1} \frac{1}{R_2 C} + V_0}$

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_e}{R_1} \frac{1}{R_2 C} \int_0^t V_e dt + V_0$$

$\therefore \quad \cancel{V_{out} = G_p V_e + G_p G I \int}$

$$\therefore V_{out} = G_p V_e + G_p G I \int_0^t V_e dt + V_{(0)}$$

Where;

$$G_P = \frac{R_2}{R_1} \text{ - - - - Proportional Gain}$$

$$G_I = \frac{1}{R_2 C} \text{ - - - - Integral Gain}$$

For Proportional Derivative;

$$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}\left[V_e - V_a\right] - \frac{V_a}{R_1} = 0 \text{ - - - equ (iii)}$$

$$\frac{V_{out}}{R_2} + \frac{V_a}{R_1} = 0 \text{ - - - - equ (iv)}$$

From equ (iv)

$$V_a = -\frac{R_1}{R_2} V_{out} \text{ - - - - equ (v)}$$

Now, Substitute equ (v) into equ (iii)

$$\frac{V_e}{R_3} - \left[\frac{-R_1}{R_2} V_{out}\right]\frac{1}{R_3} + C\frac{d}{dt} V_e - C\frac{d}{dt}\left[\frac{-R_1}{R_2} V_{out}\right] - \left[\frac{-R}{R_2} V_{out}\right]\frac{1}{R} = 0$$

$$\frac{V_e}{R_3} + \frac{R_1}{R_2 R_3} V_{out} + C\frac{d}{dt} V_e + \frac{R_1}{R_2} C \frac{d}{dt} V_{out} + \frac{1}{R_2} V_{out} = 0$$

Multiply through by R_3

$$V_e + \frac{R_1}{R_2} V_{out} + R_3 C\frac{d}{dt} V_e + \frac{R_1 R_3}{R_2} C\frac{d}{dt} V_{out} + \frac{R_3}{R_2} V_{out} = 0$$

$$\frac{R_1}{R_2} V_{out} + \frac{R_1 R_3}{R_2} C\frac{d}{dt} V_{out} + \frac{R_3}{R_2} V_{out} = -V_e - R_3 C\frac{d}{dt} V_e$$

$$\frac{R_1 V_{out} + R_3 V_{out}}{R_2} + \frac{R_1 R_3}{R_2} C\frac{d}{dt} V_{out} = -V_e - R_3 C\frac{d}{dt} V_e$$

$$\left[\frac{R_1 + R_3}{R_2}\right] V_{out} + \frac{R_1 R_3}{R_2} C\frac{d}{dt} V_{out} = -V_e - R_3 C\frac{d}{dt} V_e$$

Multiply through by $\frac{R_2}{R_1+R_3}$

$$V_{out} + \left[\frac{R_1}{R_1+R_3}\right] R_3 C \frac{d}{dt} V_{out} = -\left[\frac{R_2}{R_1+R_3}\right] V_e - \left[\frac{R_2}{R_1+R_3}\right] R_3 C \frac{d}{dt} V_e$$

Second Inversion takes place ~

$$V_{out} + \left[\frac{R_1}{R_1+R_3}\right] R_3 C \frac{d}{dt} V_{out} = \left[\frac{R_2}{R_1+R_3}\right] V_e + \left[\frac{R_2}{R_1+R_3}\right] R_3 C \frac{d}{dt} V_e$$

$$\therefore V_{out} = \left[\frac{R_2}{R_1+R_3}\right] V_e + \left[\frac{R_2}{R_1+R_3}\right] R_3 C \frac{d}{dt} V_e + V_0$$

$$V_{out} = G_P V_e + G_P G_D \frac{d}{dt} V_e + V_0$$

Where; $G_P = \frac{R_2}{R_1+R_3}$ — — — — Proportional Gain

$G_D = R_3 C$ — — — — Derivative Gain.