16/ENG07/019 Olamilola

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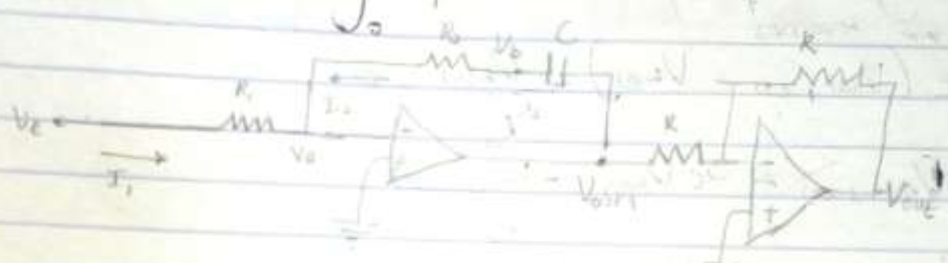
Question
Drive the analysis for the output voltage in using operational Amplifier:
(1) Proportional Integral controller Mode
(2) Proportional Derivative Controller mode

Solution
For Proportional Integral Controller Mode

$$P = k_p e_p + k_p k_I \int_0^t e_p dt + PI(0)$$



NB There is no current through the op-amp input terminals, and no Voltage across the input terminals

$V_a = 0$

$I_1 + I_2 = 0$

$I_3 - I_2 = 0$

The relationship between the voltage across the capacitor and the c through a capacitor:

$$I_c = C\frac{dV_c}{dt}$$

where $I_1 = \frac{V_e - V_a}{R_1} = \frac{V_e - 0}{R_1} = \frac{V_e}{R_1}$

$$I_1 = \frac{V_e}{R_1}$$

$I_2 = \frac{V_b - V_a}{R_2} = \frac{V_b - 0}{R_2} = \frac{V_b}{R_2}$ | $I_3 = C\frac{d}{dt}(V_{out} - V_b)$

Considering ohms low

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \quad \text{.....*}$$

$$C\frac{d}{dt}(V_{out} - V_b) - \frac{V_b}{R_2} = 0 \quad \text{...**}$$

$$\Rightarrow \frac{V_b}{R_2} = -\frac{V_e}{R_1}$$

making V_b subject of formula

$$V_b = -\frac{R_2}{R_1} V_e \quad \text{....***}$$

making use of laplace transform in s-domain equ** becomes

$$sC(V_{out,(s)} - V_{b(s)}) - \frac{V_{b(s)}}{R_2} = 0$$

$$sC\,V_{out,(s)} - sC\,V_{b(s)} - \frac{V_{b(s)}}{R_2} = 0$$

$$sC\,V_{out,(s)} = sC\,V_{b(s)} + \frac{V_{b(s)}}{R_2}$$

$$sC\,V_{out1(s)} = V_{b(s)}\left(sC + \frac{1}{R_2}\right) \quad \text{....****}$$

Substitute from equ *** into equ **** in s-domain

$$sC\,V_{out1(s)} = \frac{-R_2}{R_1} V_{e(s)}\left(sC + \frac{1}{R_2}\right)$$

$$V_{out1(s)} = \frac{-R_2}{sCR_1} V_{e(s)}\left(sC + \frac{1}{R_2}\right)$$

$$V_{out1(s)} = -\frac{R_2}{R_1} V_{e(s)} - \frac{R_2}{R_1} \cdot \frac{1}{sCR_2} V_{e(s)}$$

From the inverting circuit in the op-amp

$$V_{out1} = -V_{out}$$

$$\therefore V_{out}(s) = -\left(-\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{sCR_2} V_e(s)\right)$$

$$V_{out}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{sCR_2} V_e(s) \quad \ldots ****$$

Substituting **** using inverse laplace transform

$$V_{out} = \frac{R_2}{R_1} V_e(t) + \frac{R_2}{R_1}\frac{1}{R_2}\int_0^t V_e(t)\,dt + V_{(0)}$$

$$V_{out} = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1} \cdot \frac{1}{R_2}\int_0^t V_e(t)\,dt + V_{(0)}.$$

Inverse laplace transform s^{-1} can be expressed as $\int_0^t dt + c$

$$s^{-1} = \int_0^t dt + c \quad - *****$$

$$\therefore V_{out} = G_p V_e + G_p G_I \int_0^t V_e\,dt + V_{(0)}$$

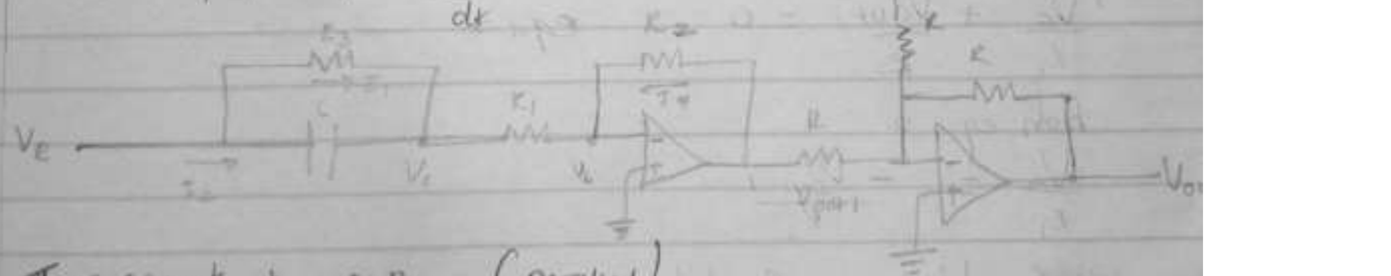
where $G_p = \frac{R_2}{R_1}$

$$G_I = \frac{1}{R_2 C}.$$

$$\therefore V_{out} = G_p V_e + G_p G_I \int_0^t V_e\,dt + V_{(0)}$$

(2) For the Proportional Derivative Controller Mode.

$$P(t) = k_p e_p + k_p K_D \frac{d e_p}{dt} + P(0)$$



From circuit $R = \frac{R_1 R_3}{R_1 + R_3}$ (parallel)

where R is the effective Resistance

[2n]/ram $R_c = 0.1$

Using KCl - PID controller

$I_1 + I_2 - I_3 = 0$

$I_1 + I_2 = I_3$

$I_4 + I_3 = 0$ / $I_3 + I_4 = 0$

where $I_1 = \frac{V_e - V_a}{R_3}$

$I_2 = \frac{Cd}{dt}(V_e - V_a)$

$I_3 = \frac{V_a - V_b}{R_1} = \frac{V_a - 0}{R_1} = \frac{V_a}{R_1}$ $(V_b = 0)$

$I_4 = \frac{V_{out1} - V_b}{R_2}$ $(V_b = 0)$

$I_4 = \frac{V_{out1}}{R_2} - 0 = \frac{V_{out1}}{R_2}$

Combining ohms law

$\frac{V_e - V_a}{R_3} + \frac{Cd}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$

$\frac{V_e - V_a}{R_3} + \frac{Cd}{dt}(V_e - V_a) = \frac{V_a}{R_1}$ ... equ 1

$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0$ ... equ 2

From equ 2

$\frac{V_a}{R_1} = -\frac{V_{out1}}{R_2}$

where $V_a = -\frac{R_1}{R_2} V_{out1}$ ——— equ 3

$$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$$

~~To~~ Replacing with laplace

$$\frac{V_{e(s)} - V_{a(s)}}{R_3} + sC\,(V_{e(s)} - V_{a(s)}) - \frac{V_{a(s)}}{R_1} = 0$$

~~[illegible]~~ rearranging equations

$$\frac{V_{e(s)}}{R_3} + sC\,V_{e(s)} = \frac{V_{a(s)}}{R_1} + \frac{V_{a(s)}}{R_3} + sC\,V_{a(s)}$$

Replacing V_a in equ 3.

$$V_{e(s)}\left(\frac{1}{R_3} + sC\right) = V_{a(s)}\left(\frac{1}{R_1} + \frac{1}{R_3} + sC\right)$$

$$V_{e(s)}\left(\frac{1 + R_3 sC}{R_3}\right) = -\frac{R_1}{R_2}V_{out_1(s)}\left(\frac{R_3 + R_1 + sCR_1R_3}{R_1R_3}\right)$$

$$V_{e(s)}(1 + sCR_3) = -\frac{R_1}{R_2}V_{out_1(s)}\left(\frac{R_3 + R_1 + sC}{R_1 \cancel{R_3}}\right)$$

$$V_{e(s)}(1 + sCR_3) = -\frac{V_{out_1(s)}}{R_2}\left(R_3 + R_1 + sCR_1R_3\right)$$

$$-V_{out_1(s)} = \frac{V_{e(s)}(1 + sCR_3)R_2}{(R_1 + R_3 + sCR_1R_3)}$$

$$-V_{out_1(s)} = \frac{V_{e(s)}(R_2 + sCR_2R_3)}{(R_1 + R_3 + sCR_1R_3)}$$

Divide all through by $R_1 + R_3$

$$-V_{out_1(s)} = \frac{V_{e(s)}(R_2 + sCR_2R_3)/R_1 + R_3}{\frac{R_1 + R_3}{R_1 + R_3} + \frac{sCR_1R_3}{R_1R_3}}$$

Parallel circuit $\cdot R = R_1R_3$

Replacing

$$-V_{out1(s)} = \frac{V_{e(s)}\left(R_2 + sCR_2R_3\right) / R_1+R_3}{1+sCR}$$

When $sCR' \ll 1$

$$-V_{out1(s)} = \frac{V_{e(s)}\left(R_2 + sCR_2R_3\right)}{R_1+R_2}$$

Applying the inverting circuit from the op-amp

$$V_{out1} = -V_{out} + V_0$$

$$-\left(\left[-V_{out(s)} + V_{0(s)}\right]\right) = \frac{V_e\left(R_2 + sCR_2R_3\right)}{R_1+R_3}$$

$$V_{out(s)} - V_{0(s)} = \frac{V_{e(s)}R_2}{R_1+R_3} + \frac{sCR_2R_3}{R_1+R_3}V_{e(s)}$$

$$V_{out(s)} = \frac{R_2}{R_1+R_3}V_{e(s)} + \frac{R_2}{R_1+R_3}R_3sC\,V_{e(s)} + V_{0(s)}$$

Applying inverse laplace transform

$$V_{out(t)} = \frac{R_2}{R_1+R_3}V_e + \frac{R_2}{R_1+R_3}R_3C\frac{dV_e}{dt} + V_0$$

$$V_{out(t)} = G_pV_e + G_pG_D\frac{dV_e}{dt} + V_0$$

$$V_{out} = G_pV_e + G_pG_D\frac{dV_e}{dt} + V_0$$

recall

$$\therefore G_p = \frac{R_2}{R_1+R_3}$$

$$G_D = R_3C$$