Angon Nsikan
16/ENG05/009
Mechatronics Engineering
MCT 511

Assignment

1)

$$\frac{Cd}{dt}[V_{out} - V_b] - \frac{V_b}{R_2} = 0 \quad -(1)$$

$$\left.\frac{V_e - V_c}{R_3} + \frac{Cd}{dt}[V_2 - V_1] - \frac{V_a}{R_1} = 0 \right\}(2)$$

PI

$$\frac{Cd}{dt}[V_{out} - V_b] = \frac{V_b}{R_2}$$

$$\frac{d}{dt}[V_{out} - V_b] = \frac{V_b}{R_2 C}$$

Recall

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0$$

$$\frac{V_b}{R_2} = \frac{V_e}{R_1}$$

$$V_b = \frac{R_2}{R_1} V_e \quad -(1)$$

$$\int d[V_{out} - V_b] = \int \frac{V_b}{R_2 C} dt$$

$$V_{out} - V_b = \int \frac{V_b}{R_2 C} dt$$

$$V_{out} = \int \frac{V_b}{R_2 C} dt + V_b$$

$$= \frac{1}{R_2 C} \int \frac{R_2}{R_1} V_e dt + \left[-\frac{R_2}{R_1} V_e \right]$$

$$= \frac{1}{R_2 C} \frac{R}{R_1} \int_1^t V_e dt - \frac{R_2}{R_1} V_e$$

$$= -\left[\frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \frac{1}{R_2 C} \int_0^t V_e \, dt \right] + V_{(0)}$$

After Inverting

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_e}{R_1} \frac{1}{R_2 C} \int_0^t V_e dt + V_{(0)}$$

$$V_{out} = G_p V_e + G_e G_I \int_0^t V_e dt + V_{(0)}$$

Where

$G_p = \frac{R_2}{R_1}$ Proportional Gain

$G_I = \frac{1}{R_2 C}$ Integral Gain.

2) $\frac{V_e - V_o}{R_3} + C \frac{d}{dt}[V_e - V_o] - \frac{V_a}{R_1} = 0$ --- *

$\frac{V_{out}}{R_2} + \frac{V_a}{R_1} = 0$ --- **

Then from **

$$V_a = -\frac{R_1}{R_2} V_{out}$$

Substituting into equ *

$$\frac{V_e}{R_3} - \left[-\frac{R_1}{R_2} V_{out}\right]\frac{1}{R_3} + C\frac{dV_e}{dt} - C\frac{d}{dt}\left[-\frac{R_1}{R_2} V_{out}\right] + \left[\frac{-R_1 V_{out}}{R_2}\right] \times \frac{1}{R}$$

$$\frac{V_e}{R_3} + \frac{R_1}{R_2 R_3} V_{out} + C\frac{dV_e}{dt} + \frac{R_1}{R_2} C \frac{d}{dt} V_{out} + \frac{1}{R_2} V_{out} = 0$$

Multiply through by R_3

$$V_e + \frac{R_1}{R_2} V_{out} + R_3 C \frac{dV_e}{dt} + \frac{R_1 R_3}{R_2} C \frac{dV_{out}}{dt} + \frac{R_3}{R_2} V_{out} = 0$$

$$\frac{R_1}{R_2} V_{out} + \frac{R_1 R_3}{R_2} C \frac{dV_{out}}{dt} + \frac{R_3}{R_2} V_{out} = -V_e - R_e C \frac{dV_e}{dt}$$

$$\frac{R_3 V_{out} + R_3 V_{out}}{R_2} + \frac{R_1 R_2}{R_2} C \frac{dV_{out}}{dt} = -V_e - R_e C \frac{dV_e}{dt}$$

$$\frac{R_1 V_{out} + R_3 V_{out}}{R_2} + \frac{R_1 R_3}{R_2} C \frac{dV_{out}}{dt} = -V_e - R_e C \frac{dV_e}{dt}$$

$$\left[\frac{R_1 + R_3}{R_2}\right] V_{out} + \frac{R_1 R_3}{R_2} C \frac{dV_{out}}{dt} = -V_e - R_3 C \frac{dV_e}{dt}$$

Multiply through by $\frac{R_2}{R_1 + R_3}$

$$V_{out} + \left[\frac{R_1}{R_1 + R_3}\right] R_3 C \frac{dV_{out}}{dt} = -\left[\frac{R_2}{R_1 + R_3}\right] V_e - \left[\frac{R_2}{R_1 + R_3}\right] R_3 C \frac{dV_e}{dt}$$

After Inverting

$$V_{out} + \left[\frac{R_1}{R_1 + R_3}\right] R_3 C \frac{dV_{out}}{dt} = \left[\frac{R_2}{R_1 + R_3}\right] V_e + \left(\frac{R_2}{R_1 + R_3}\right) R_3 C \frac{dV_e}{dt}$$

$\therefore$

$$V_{out} = \left(\frac{R_2}{R_1+R_3}\right)V_e + \left(\frac{R_2}{R_1+R_3}\right)R_3 C \frac{dV_e}{dt} + V(0)$$

$$V_{out} = G_p V_e + G_p G_D \frac{dV_e}{dt} + V_0$$

Where

$G_p = \frac{R_2}{R_1+R_3}$ - Proportional gain

$G_D = R_3 C$ - Derivative gain