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Elect/ Elect Engineering

EEE 441. SERVOMECHANISM AND CONTROL SYSTEMS

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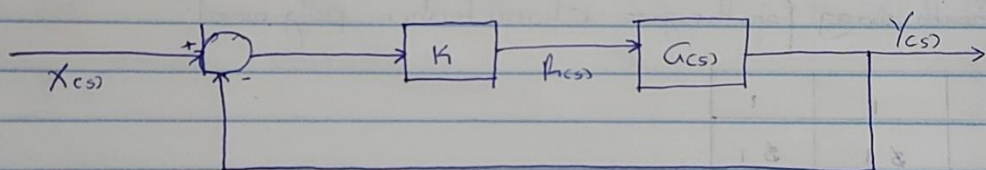
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EEE 441. Servo Mechanism & Control Systems.

1) Briefly Explain the Root Locus Technique.

The Root Locus is a graphical technique by means of which the locus (or variation) of the roots of the characteristic equation of the closed-loop system produced by the variation of some system parameter - usually the system gain k - can be deduced from knowledge of the open loop system together with a set of rules for constructing the locus. From this locus it is possible to choose a particular value of k that is likely to result in reasonable stability, & it is then quite simple to obtain the closed-loop transient response. This is a method of factoring the closed loop transfer function.



Closed loop system considered above with unity feedback that uses simple proportional controller. It has a TF

$$H(s) = \frac{kG(s)}{1+kG(s)} = \frac{N(s)}{D(s)}$$

The root locus is the locus of the roots of characteristic equation by varying k from zero \rightarrow infinity.

The poles occur at the roots $D(s)$. for the system TF above these roots occur where

$$1 + kG(s) = 0.$$

This is referred to as the characteristic equation of the system, therefore it is necessary that

$$\angle |kG(s)| = 1$$

$$\angle G(s) = 180^\circ \pm k360 \text{ for } k \in \mathbb{Z}$$

The 2 equations are referred to as the Magnitude & Angle Criteria respectively.
 ∴ The root Locus is the path of the roots of the characteristic equation due to K as K is traced out to infinity.

2a) Considering this Example
 Find the stability of the control System having characteristic Equations

$$S^5 + 3S^4 + S^3 + 3S^2 + S + 3 = 0$$

Step

1] Verify the necessary condition for the Routh-Hurwitz stability.

All the coefficients of the given characteristic polynomial are positive. So the control System satisfied the necessary condition.

Step 2: Form the Routh array for the given characteristic polynomial.

S^5	1	1	1
S^4	3	3	3
S^3	$(1 \times 1) - (1 \times 1)$ $= 0$	$(1 \times 1) - (1 \times 1)$ $= 0$	
S^2			
S^1			
S^0			

The row S^4 elements have the common factor of 3. So, all these elements are divided by 3.

- All the elements of row S^3 are zero. So, write the auxiliary equation, $A(s)$ of the row S^4

$$A(s) = S^4 + S^2 + 1$$

Differentiate the above equation with respect to s

$$\frac{dA(s)}{ds} = 4S^3 + 2S$$

s^5	1	1	1
s^4	1	1	1
s^3	4 2	2 1	
s^2	$\frac{(2 \times 1) - (1 \times 1)}{2}$ $= 0.5$	$\frac{2 \times 1 - (1 \times 1)}{1}$ $= 1$	
s^1	$\frac{(0.5 \times 1) - (1 \times 2)}{0.5}$ $= \frac{-1.5}{0.5}$ $= -3$		
s^0	1		

Step 3 - Verify the Sufficient condition for the Routh-Hurwitz stability. There are 2 sign changes in the 1st column of Routh table. Hence, the control system is unstable.

b] A whole row of zeros indicates the presence of pairs of poles that are mirrored about the imaginary axis (jw axis)

