MAC-ETELI GOLDEN
16/ENG05/021

$$C\frac{d}{dt}\left[V_{out} - V_b\right] - \frac{V_b}{R_2} = 0 \quad \text{——} \quad (1) \quad PI$$

$$\left.\begin{aligned} &\frac{V_c - V_a}{R_2} + C\frac{d}{dt}\left[V_c - V_a\right] - \frac{V_a}{R_1} = 0 \\ &\frac{V_{out}}{R_2} + \frac{V_o}{R_1} = 0 \end{aligned}\right\} (2)$$

For PI

$$C\frac{d}{dt}\left[V_{out} - V_b\right] - \frac{V_b}{R_2} = 0$$

$$C\frac{d}{dt}\left[V_{out} - V_b\right] = \frac{V_b}{R_2}$$

$$\frac{d}{dt}\left[V_{out} - V_b\right] = \frac{V_b}{CR_2}$$

Recall

$$\frac{V_c}{R_1} + \frac{V_b}{R_2} = 0$$

$$\frac{V_b}{R_2} = -\frac{V_c}{R_1}$$

$$V_b = -\frac{R_2}{R_1} V_c$$

$$\int d\left[V_{out} - V_b\right] = \int \frac{V_b}{CR_2}\, dt$$

$$V_{out} - V_b = \int \frac{V_b}{CR_2}\, dt$$

$$V_{out} = \int \frac{V_b}{R_2 C}\, dt + V_b$$

$$= \frac{1}{R_2 C}\int -\frac{R_2}{R_1} V_c\, dt + \left[-\frac{R_2}{R_1} V_c\right]$$

$$= -\frac{1}{R_2 C} \cdot \frac{R}{R_1}\int V_c\, dt - \frac{R_2}{R_1} V_c$$

$$= -\left[\frac{R_2}{R_1} V_c + \frac{R_2}{R_1}\frac{1}{R_2 C}\int V_c\, dt + V(0)\right.$$

After Inverting

$$V_{out} = \frac{R_2}{R_1} V_c + \frac{R_2}{R_1}\frac{1}{R_2 C}\int V_c\,dt + V(0)$$

$$V_{out} = G_P V_c + G_P G_I \int V_c\,dt + V(0)$$

where

$G_P = \frac{R_2}{R_1}$ Proportional Gain

$G_I = \frac{1}{R_2 C}$ Integral Gain

For PD

$$\frac{V_c - V_a}{R_3} + C\frac{d}{dt}\left[V_c - V_a\right] - \frac{V_a}{R_1} = 0 \quad \text{———} \ *$$

$$\frac{V_{out}}{R_2} + \frac{V_a}{R_1} = 0 \quad \text{———} \ **$$

From **

$$V_a = -\frac{R_1}{R_2} V_{out}$$

Substituting into eqn *

$$\frac{V_c}{R_3} - \left[-\frac{R_1}{R_2}V_{out}\right]\frac{1}{R_3} + C\frac{dV_c}{dt} - C\frac{d}{dt}\left[-\frac{R_1}{R_2}V_{out}\right] - \left[-\frac{R_1}{R_2}V_{out}\right]\frac{1}{R} = 0$$

$$\frac{V_c}{R_3} + \frac{R_1}{R_2R_3}V_{out} + C\frac{dV_c}{dt} + \frac{R_1}{R_2}C\frac{d}{dt}V_{out} + \frac{1}{R_2}V_{out} = 0$$

Multiply all through by R_3

$$V_c + \frac{R_1}{R_2}V_{out} + R_3C\frac{dV_c}{dt} + \frac{R_1R_3}{R_2}C\frac{dV_{out}}{dt} + \frac{R_1}{R_2}V_{out} = 0$$

$$\frac{R_1}{R_2}V_{out} + \frac{R_1R_3}{R_2}C\frac{dV_{out}}{dt} + \frac{R_3}{R_2}V_{out} = -V_c - R_3C\frac{dV_c}{dt}$$

$$\frac{R_1 V_{out} + R_3 V_{out}}{R_2} + \frac{R_1 R_3}{R_2} C \frac{dV_{out}}{dt} = -V_c - R_3 C \frac{dV_c}{dt}$$

$$\left[\frac{R_1 + R_3}{R_2}\right] V_{out} + \frac{R_1 R_3}{R_2} C \frac{dV_{out}}{dt} = -V_c - R_3 C \frac{dV_c}{dt}$$

Multiply all through by $\frac{R_2}{R_1 + R_3}$

$$V_{out} + \left[\frac{R_1}{R_1 + R_3}\right] R_3 C \frac{dV_{out}}{dt} = -\left[\frac{R_2}{R_1 + R_3}\right] V_c - \left[\frac{R_1}{R_1 + R_3}\right] R_3 C \frac{dV_c}{dt}$$

After inverting

$$V_{out} + \left[\frac{R_1}{R_1 + R_3}\right] R_3 C \frac{dV_{out}}{dt} = \left[\frac{R_2}{R_1 + R_3}\right] V_c + \left[\frac{R_2}{R_1 + R_3}\right] R_3 C \frac{dV_c}{dt}$$

$$V_{out} = \left[\frac{R_2}{R_1 + R_3}\right] V_c + \left[\frac{R_2}{R_1 + R_3}\right] R_3 C \frac{dV_c}{dt} + V(0)$$

$$V_{out} = G_p V_c + G_p G_I \frac{dV_c}{dt} + V_0$$

where

$$G_p = \frac{R_2}{R_1 + R_3}$$ — Proportional Gain

$$G_p = R_3 C$$ — Derivative Gain