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16/ENG04/042

EEE 561 - PROCESS CONTROL & AUTOMATION

PI controller

$V_a = 0$

$I_1 + I_2 = 0$ — ①

$I_2 - I_3 = 0$ — ②

Current flowing through a capacitor: $I_c = C\frac{dV}{dt}$

$I_1 = \frac{V_c - V_a}{R_1}$ $(V_a = 0)$ $= \frac{V_c}{R_1}$

$I_2 = \frac{V_b - V_a}{R_2}$ $(V_a = 0)$ $= \frac{V_b}{R_2}$

$I_3 = C\frac{d}{dt}(V_{out} - V_b)$

sub into eq ① & ②

$\frac{V_c}{R_1} + \frac{V_b}{R_2} = 0$ — ③

$C\frac{d}{dt}(V_{out} - V_b) - \frac{V_b}{R_2} = 0$ — ④

from eq ③

$\frac{V_b}{R_2} = -\frac{V_c}{R_1}$

$V_b = -\frac{R_2 V_c}{R_1}$

Taking the laplace transform of eq ④

$SC(V_{out_1(s)} - V_{b(s)}) - \frac{V_{b(s)}}{R_2} = 0$

$SC(V_{out_1(s)}) = SCV_{b(s)} + \frac{V_{b(s)}}{R_2}$

$SC\,V_{out_1(s)} = V_{b(s)}\left(SC + \frac{1}{R_2}\right)$

Recall, $V_b = \frac{-R_2}{R_1}V_c$

$$SC\, V_{out\,I}(s) = -\frac{R_2}{R_1} V_e(s) \left(SC + \frac{1}{R_2}\right)$$

$$V_{out\,I}(s) = \frac{-R_2}{SCR_1} V_e(s) \left(SC + \frac{1}{R_2}\right)$$

$$V_{out\,I}(s) = -\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)$$

from the inverting circuit

$$V_{out\,I} = -V_{out}$$

$$\therefore V_{out}(s) = -\left(-\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)\right)$$

$$V_{out}(s) = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)$$

Taking the inverse laplace

$$V_{out} = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{R_2}\int_0^t V_e(t)\,dt + V_{(0)}$$

$$\text{where } \frac{1}{s} = \int_0^t dt + K$$

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e\,dt + V_{(0)}$$

$$\text{where } G_P = \frac{R_2}{R_1}$$

$$G_I = \frac{1}{R_2 C}$$

PD controller

$I_1 + I_2 = I_3$ —①

$I_3 + I_4 = 0$ —②

$I_1 = \frac{V_e - V_a}{R_3}$

$I_2 = C\frac{d}{dt}(V_e - V_a)$

$I_3 = \frac{V_a - V_b}{R_1}$ $(V_b = 0)$ $= \frac{V_a}{R_1}$

$I_4 = \frac{V_{out1} - V_b}{R_2}$ $(V_b = 0)$ $= \frac{V_{out1}}{R_2}$

$R = \frac{R_1 R_3}{R_1 + R_3}$ where R = Effective Resistance

Sub into eq① & eq②

$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}(V_e - V_a) = \frac{V_a}{R_1}$ —③

$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0$ —④

from eq ④

$\frac{V_a}{R_1} = -\frac{V_{out1}}{R_2}$; $V_a = -\frac{R_1}{R_2} V_{out1}$

from eq (3)

$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$

Using the laplace transform

$\frac{V_e(s) - V_a(s)}{R_3} + SC(V_e(s) - V_a(s)) - \frac{V_a(s)}{R_1} = 0$

(Initial conditions $\to 0$)

$\frac{V_e(s)}{R_3} + SC V_e(s) = \frac{V_a(s)}{R_1} + \frac{V_a(s)}{R_3} + SC V_a(s)$

$V_e(s)\left(\frac{1}{R_3} + SC\right) = V_a(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + SC\right)$

Recall; $V_a = -\frac{R_1}{R_2} V_{out1}$

$V_e(s)\cdot\left(\frac{1}{R_3} + SC\right) = \frac{-R_1}{R_2} V_{out1}(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + SC\right)$

Taking LCM

$$V_{e(s)}\left(\frac{1+R_3SC}{R_3}\right) = -\frac{R_1}{R_2}V_{out1(s)}\left(\frac{R_3+R_1+SCR_1R_3}{R_1R_3}\right)$$

$$V_{e(s)}(1+SCR_3) = -\frac{V_{out1(s)}}{R_2}(R_3+R_1+SCR_1R_3)$$

$$-V_{out1(s)} = \frac{V_{e(s)}(1+SCR_3)R_2}{(R_1+R_3+SCR_1R_3)}$$

$$-V_{out1(s)} = \frac{V_{e(s)}(R_2+SCR_2R_3)}{(R_1+R_3+SCR_1R_3)}$$

Dividing numerator by denominator by R_1+R_3

$$-V_{out1(s)} = \frac{V_{e(s)}(R_2+SCR_2R_3)/R_1+R_3}{\frac{R_1+R_3}{R_1+R_3}+\frac{SCR_1R_3}{R_1+R_3}}$$

Recall, $R = \frac{R_1R_3}{R_1+R_3}$

$$-V_{out1(s)} = \frac{V_{e(s)}(R_2+SCR_2R_3)/R_1+R_3}{1+SCR}$$

if $SCR << 1$

$$-V_{out(s)} = \frac{V_{e(s)}(R_2+SCR_2R_3)}{R_1+R_3}$$

from the inverting circuit

$$V_{out1} = -V_{out} + V_0$$

$$\therefore -(-V_{out(s)}+V_{0}) = \frac{V_e(R_2+SCR_2R_3)}{R_1+R_3}$$

$$V_{out(s)} - V_{0} = \frac{V_{e(s)}R_2}{R_1+R_3} + \frac{SCR_2R_3}{R_1+R_3}V_{e(s)}$$

$$V_{out(s)} = \frac{R_2}{R_1+R_3}V_{e(s)} + \frac{R_2}{R_1+R_3}R_3SCV_{e(s)} + V_{(0)}$$

Taking inverse laplace

$$V_{out} = \frac{R_2}{R_1+R_3}V_e + \frac{R_2}{R_1+R_3}R_3C\frac{dV_e}{dt} + V_0$$

$$V_{out} = G_pV_e + G_pG_D\frac{dV_e}{dt} + V_0$$

where, $G_p = \frac{R_2}{R_1+R_3}$

$$G_D = R_3C$$