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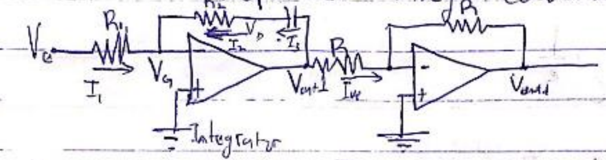
16/ENG04/004

Assignment Solution

Proportional Integral controller mode



$V_{out} = \frac{-R}{R}(V_{out}1) = -V_{out}1$ [Inverting Circuit]

$V_a = 0$

Applying KCL to the Integrator Op amp

$I_1 + I_2 = 0$ ----(1)

$I_3 - I_2 = 0$ ---- (2)

Note: $I_2 = I_c = C\frac{dV_c}{dt} = \frac{C\,d(V_{out}1 - V_b)}{dt}$ ...(3)

Applying ohm's law to equation (1) and (2).

$\frac{V_a}{R_1} + \frac{V_b}{R_2} = 0$ ---- (4)

$\therefore V_b = -\frac{R_2}{R_1}V_e$ .... (5)

$C\frac{d}{dt}(V_{out}1 - V_b) - \frac{V_b}{R_2} = 0$ .... (6)

By expansion, eqn(6)

$C\frac{dV_{out}1}{dt} - \frac{C\,dV_b}{dt} - \frac{V_b}{R_2} = 0$

divide through by C

$\frac{dV_{out}1}{dt} - \frac{dV_b}{dt} - \frac{V_b}{R_2C} = 0$ ..... (7)

Take the laplace of eqn(7)

$SV_{out}1 - SV_b - \frac{V_b}{R_2C} = 0$

$$V_{out} = V_b + \frac{V_b}{R_2 C} \cdot \frac{1}{s}$$

$V_b = -\frac{R_2}{R_1} V_e$, eqn(1) below

$$\cancel{V_{out1} = V_b + \frac{V_b}{R_2 C} \cdot \frac{1}{s}}$$

~~By substituting~~ $\cancel{V_b = -\frac{R_2}{R_1} V_e}$

$$V_{out1} = -\frac{R_2}{R_1} V_e - \frac{R_2}{R_1} \cdot \frac{1}{R_2 C} V_e \cdot \frac{1}{s}$$

Also, $V_{out} = -V_{out1}$

$$\therefore -V_{out} = -\frac{R_2}{R_1} V_e - \frac{R_2}{R_1} \cdot \frac{1}{R_2 C} V_e \cdot \frac{1}{s}$$

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot \frac{1}{R_2 C} V_e \frac{1}{s}$$

Taking inverse laplace transform

$\frac{1}{s} = \int_0^t dt + k$, k = Integration constant

Also, applying the integrator circuit analysis principle:

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot \frac{1}{R_2 C} \int_0^t V_e \, dt + V_{out}(0)$$

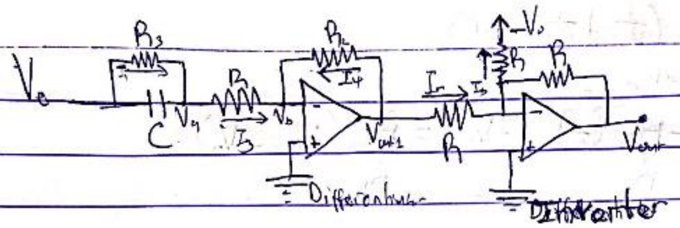
$$V_{out} = G_p V_e + G_p G_I \int_0^t V_e \, dt + V_{out}(0)$$

$V_{out}(0)$ = Output Voltage with no error Voltage

$G_p = \frac{R_2}{R_1}$ [Proportional gain]

$G_I = \frac{1}{R_2 C}$ [Integral gain]

Proportional Derivative Controller Mode



$$R = \frac{R_1 R_3}{R_1 + R_3}$$

R is effective resistance

Then, the condition become as usual: $2\pi f_{max} RC = 0.1$

By KCL

$I_1 + I_2 - I_3 = 0$ ...... (1)

$I_4 + I_3 = 0$ ...... (2)

Combining with ohms law

$$\frac{V_e - V_a}{R_3} + \frac{C\, d(V_e - V_a)}{dt} - \frac{V_a}{R_1} = 0 \quad \text{...... (3)}$$

$$\frac{V_{out1}}{R_2} + \frac{V_a}{R_1} = 0 \quad \text{...... (4)}$$

$$V_a = -\frac{R_1}{R_2} V_{out1} \quad \text{...... (5)}$$

Also: $I_5 + I_7 - I_6 = 0$ ...... (6)

$$\frac{V_{out1}}{R} + \frac{V_{out}}{R} - \frac{V_o}{R} = 0 \quad \text{...... (7)}$$

$\therefore V_{out} = -V_{out1} + V_o$ ...... (8)

By expansing eq (3) below:

$$\frac{V_e}{R_3} + \frac{C\,dV_e}{dt} = \frac{V_a}{R_3} + \frac{V_a}{R_1} + \frac{C\,dV_a}{dt} \quad \text{... (9)}$$

$$V_o\left(\frac{1}{R_3}+SC\right)=V_a\left(\frac{1}{R_3}+\frac{1}{R_1}+SC\right)$$

By substitution

$$V_a=-\frac{R_1}{R_2}V_{out}1=-\frac{R_1}{R_2}(-V_{out}+V_o)=\frac{R_1}{R_2}(V_{out}-V_o)$$

equation becomes:

$$\left(\frac{R_1}{R_2}\right)(V_{out}-V_o)\left(\frac{1}{R_3}+\frac{1}{R_1}+SC\right)=V_o\,\frac{(1+R_3SC)}{R_3}$$

$$\left(\frac{R_1}{R_2}\right)(V_{out}-V_o)\frac{(R_1+R_3+R_1R_3SC)}{R_3R_1}=V_o\frac{(1+R_3SC)}{R_3}$$

$$V_{out}-V_o=\frac{V_o(1+R_3SC)\times R_2}{R_1+R_3+R_1R_3SC}$$

Divide both numerator & denominator by R_1+R

$$V_{out}-V_o=V_o\frac{\left(\frac{1}{R_1+R_3}+\frac{R_3SC}{R_1+R_3}\right)R_2}{1+\frac{R_1R_3SC}{R_1+R_3}}$$

$$V_{out}-V_o=\frac{\left(\frac{R_2}{R_1+R_3}+\frac{R_2R_3SC}{R_1+R_3}\right)V_o}{1+RSC}$$

If $RCS<<1$,

$$V_{out}-V_o=\frac{R_2}{R_1+R_3}V_o+\frac{R_2}{R_1+R_3}\cdot R_3CSV_o$$

By taking inverse laplace transfom

$$SV_o=\frac{dV_o}{dt},\quad V_{out}=\frac{R_2}{R_1+R_3}V_o+\frac{R_2}{R_1+R_3}\cdot R_3C\frac{dV_o}{dt}+V_o$$

$$V_{out}=G_pV_o+G_pG_o\frac{dV_o}{dt}+V_o$$

$$G_p=\frac{R_2}{R_1+R_3}\longrightarrow \text{Proportional gain}$$

$$G_o=R_3C\longrightarrow \text{Derivative gain.}$$