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Proportional Integral Controller

$V_a = 0$

$I_1 + I_2 = 0$ --- ①

$I_3 - I_2 = 0$ --- ②

Current through the capacitor

$I_C = C\frac{dV_c}{dt}$

$I_1 = \frac{V_c - V_a}{R_1}$ $(V_a = 0)$

$= \frac{V_c}{R_1}$

$I_2 = \frac{V_b - V_a}{R_2}$ $(V_a = 0)$

$= \frac{V_b}{R_2}$

$I_3 = C\frac{d}{dt}(V_{out1} - V_b)$

Sub into eqn ① & eqn ②

$\frac{V_c}{R_1} + \frac{V_b}{R_2} = 0$ --- ①

$$\boxed{C\frac{d}{dt}(V_{out1} - V_b) - \frac{V_b}{R_2} = 0} \quad \text{— ②}$$

From eqn ①

$\frac{V_b}{R_2} = \frac{V_c}{R_1}$

$$\boxed{V_b = -\frac{R_2 V_c}{R_1}}$$

taking laplace transform of eqn ②

$SC(V_{out1}(s) - V_b(s)) - \frac{V_b(s)}{R_2} = 0$

$$SC\, V_{out1}(s) = SC\, V_b(s) + \frac{V_b(s)}{R_2}$$

$$SC\, V_{out1}(s) = V_b(s)\left(SC + \frac{1}{R_2}\right)$$

recall; $V_b = -\frac{R_2}{R_1} V_c$

$$SC\, V_{out1}(s) = -\frac{R_2}{R_1} V_c(s)\left(SC + \frac{1}{R_2}\right)$$

$$V_{out1}(s) = \frac{-R_2}{SCR_1} V_c(s)\left(SC + \frac{1}{R_2}\right)$$

$$V_{out1}(s) = -\frac{R_2}{R_1} V_c(s) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_c(s)$$

From the inverting circuit

$$V_{out1} = -V_{out}$$

$$\therefore V_{out}(s) = -\left(-\frac{R_2}{R_1} V_c(s) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_c(s)\right)$$

$$V_{out}(s) = \frac{R_2}{R_1} V_c(s) + \frac{R_2}{R_1}\frac{1}{SCR_2} V_c(s)$$

taking inverse laplace

$$V_{out} = \frac{R_2}{R_1} V_c(s) + \frac{R_2}{R_1}\frac{1}{R_2}\int_0^t V_c(t)\, dt + V(0)$$

$$\left(\text{where } \frac{1}{s} = \int_0^t dt + K\right)$$

$$\boxed{V_{out} = G_p V_c + G_p G_I \int_0^t V_c dt + V(0)}$$

where $G_p = \frac{R_2}{R_1}$

$$G_I = \frac{1}{R_2 C}$$

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Propoitional ~~Anyway~~ Derivative Controller

$I_1 + I_2 = I_3$ — ①

$I_3 + I_4 = 0$ — ②

$$I_1 = \frac{V_c - V_a}{R_3}$$

$$I_2 = \frac{Cd}{dt}(V_c - V_a)$$

$$I_3 = \frac{V_a - V_b}{R_1} \quad (V_b = 0)$$

$$= \frac{V_a}{R_1}$$

$$I_4 = \frac{V_{out_1} - V_b}{R_2} \quad (V_b = 0)$$

$$= \frac{V_{out_1}}{R_2}$$

$$R = \frac{R_1 R_3}{R_1 + R_3}$$ — effective resistance

Sub into eqn ① & eqn ②

$$\frac{V_c - V_a}{R_3} + \frac{Cd}{dt}(V_c - V_a) = \frac{V_a}{R_1} \quad — *$$

$$\frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0 \quad — **$$

From eqn **

$$\frac{V_a}{R_1} = -\frac{V_{out_1}}{R_2}$$

$$\boxed{V_a = -\frac{R_1}{R_2} V_{out1}}$$

rearranging eqn *

$$\frac{V_c - V_a}{R_3} + \frac{Cd}{dt}(V_c - V_a) - \frac{V_a}{R_1} = 0$$

taking Laplace transform

$$V_{c(s)}\ V_{a(s)} + SC\left(V_{c(s)} - V_{a(s)}\right) - \frac{V_{a(s)}}{R_1} = 0$$

(initial conditions go to zero)

$$\frac{V_c(s)}{R_3} + SC\,V_c(s) = \frac{V_a(s)}{R_1} + \frac{V_a(s)}{R_3} + SC\,V_a(s)$$

$$V_c(s)\left(\frac{1}{R_3} + SC\right) = V_a(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + SC\right)$$

recall, $V_a = -\frac{R_1}{R_2} V_{out_1}$

$$V_c(s)\left(\frac{1}{R_3} + SC\right) = -\frac{R_1}{R_2} V_{out_1}(s)\left(\frac{1}{R_1} + \frac{1}{R_3} + SC\right)$$

taking lim

$$V_c(s)\left(\frac{1 + R_3 SC}{\cancel{R_3}}\right) = -\frac{\cancel{R_1}}{R_2} V_{out_1}(s)\left(\frac{R_3 + R_1 + SCR_1R_3}{\cancel{R_1}\cancel{R_3}}\right)$$

$$V_c(s)\left(1 + SCR_3\right) = -\frac{V_{out_1}(s)}{R_2}\left(R_3 + R_1 + SCR_1R_3\right)$$

$$-V_{out_1}(s) = \frac{V_c(s)\left(1 + SCR_3\right)R_2}{\left(R_1 + R_3 + SCR_1R_3\right)}$$

$$-V_{out_1}(s) = \frac{V_c(s)\left(R_2 + SCR_2R_3\right)}{\left(R_1 + R_3 + SCR_1R_3\right)}$$

dividing numerators & denominators by $R_1 + R_3$

$$-V_{out_1}(s) = \frac{V_c(s)\left(R_2 + SCR_2R_3\right)/R_1 + R_3}{\frac{R_1 + R_3}{R_1 + R_3} + \frac{SCR_1R_3}{R_1 + R_3}}$$

recall, $R = \frac{R_1R_3}{R_1 + R_3}$

$$-V_{out_1}(s) = \frac{V_c(s)\left(R_2 + SCR_2R_3\right)/R_1 + R_3}{1 + SCR}$$

If $SCR \ll 1$

$$-V_{out_1}(s) = \frac{V_c(s)\left(R_2 + SCR_2R_3\right)}{R_1 + R_3}$$

from the inverting circuit

$$V_{out_1} = -V_{out} + V_c$$

$$\therefore\ -\left(-V_{out}(s) + V_c(s)\right) = \frac{V_c\left(R_2 + SCR_2R_3\right)}{R_1 + R_3}$$

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$$V_{out}(s) - V(0) = \frac{V_c(s) R_2}{R_1 + R_3} + \frac{sCR_2 R_3}{R_1 + R_3} V_c(s)$$

$$V_{out}(s) = \frac{R_2}{R_1 + R_3} V_c(s) + \frac{R_2}{R_1 + R_3} R_3 C s V_c(s) + V(s)$$

taking inverse laplace

$$V_{out} = \frac{R_2}{R_1 + R_3} V_c + \frac{R_2}{R_1 + R_3} R_3 C \frac{dV_c}{dt} + V_0$$

$$\boxed{V_{out} = G_P V_c + G_P G_D \frac{dV_c}{dt} + V_0}$$

Where; $G_P = \frac{R_2}{R_1 + R_3}$

$G_D = R_3 C$