AGBOOLA PRAISE
16/ENG07/005 MECHATRONICS

$$\frac{Cd}{dt}[V_{out} - V_2] - \frac{V_2}{R_2} = 0 \quad \text{— ① PI}$$

$$\left.\begin{array}{l} \dfrac{V_2 - V_a}{R_3} + \dfrac{Cd}{dt}[V_2 - V_{in}) - \dfrac{V_a}{R_1} = 0 \\ \dfrac{V_{out}}{R_2} 1 + \dfrac{V_0}{R_1} = 0 \end{array}\right\} 2$$

PI

$$\frac{Cd}{dt}[V_{out} - V_2] - \frac{V_b}{R_2} = 0$$

$$C\frac{d}{dt}[V_{out} - V_2] = \frac{V_b}{R_2}$$

$$\frac{d}{dt}[V_{out} - V_1) = \frac{V_b}{R_2}$$

Recall

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0$$

$$\frac{V_b}{R_2} = \frac{V_e}{R_1}$$

After Inverting

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \frac{1}{R_2 C} \int_0^t V_e \, dt + V_{(0)}$$

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e \, dt + V_{(0)}$$

Where

$G_P = \frac{R_2}{R_1}$ Proportional Gain

$G_I = \frac{1}{R_2 C}$ Integral gain

PD

$$\frac{V_e - V_a}{R_3} + \frac{Cd}{dt}[V_e - V_a] - \frac{V_a}{R_1} = 0 \quad — ①$$

$$\frac{V_{out}}{R_2} + \frac{V_a}{R_1} = 0 \quad — (ii)$$

from (ii)

$$V_a = -\frac{R_1}{R_2} V_{out}$$

Substituting into eqn ②

$$\frac{V_c}{R_3} - \left[\frac{-R_1}{R_2} V_{out}\right]\frac{1}{R_3} + C\frac{dV_c}{dt} - C\frac{d}{dt}\left[\frac{-R_1 V_{out}}{R_2}\right]\left[\frac{R_1 V_{out}}{R_2}\right]_R$$
$$= 0$$

$$\frac{V_c}{R_3} + \frac{R_1}{R_2 R_{13}} V_{out} + C\frac{dV_c}{dt} + \frac{R_1}{R_2} C\frac{dV_{out}}{dt} + \frac{1}{R_2} V_{out}$$
$$= 0$$

Multiply through by R_3

$$V_c + \frac{R_1}{R_2} V_{out} + R_3 C\frac{dV_c}{dt} + \frac{R_1 R_3}{R_2} C\frac{dV_{out}}{dt} +$$
$$\frac{R_1}{R_2} V_{out} = 0$$

$$\frac{R_1}{R_2} V_{out} + \frac{R_1 R_3}{R_2} \frac{C\,dV_{out}}{dt} + \frac{R_3 V_{out}}{R_2} = -V_c$$ ~~R_3~~
$$- R_3 C\frac{dV_c}{dt}$$

$$\frac{R_1 V_{out} + R_3 V_{out}}{R_2} + \frac{R_1 R_3}{R_2} C\frac{dV_{out}}{dt} = -V_c - R_3 C\frac{dV_t}{dt}$$

$$V_o = \frac{R_2}{R_1} V_e$$

$$\int d\left[V_{out} - V_o\right] = \int \frac{V_b}{R_2 C} \, dt$$

$$V_{out} - V_o = \int \frac{V_o}{R_2 C} \, dt$$

$$V_{out} = \int_0^t \frac{V_b}{R_2 C} \, dt + V_b$$

$$= \frac{1}{R_2 C} \int \frac{R_2}{R_1} V_e dt + \left[-\frac{R_2}{R_1} V_e\right]$$

$$= \frac{1}{R_2 C} \frac{R}{R_1} \int^t V_e \, dt - \frac{R_2}{R_1} V_e$$

$$= -\left[\frac{R_2}{R_1} V_e + \frac{R_o}{R_1} \frac{1}{R6} \int_0^t V_e dt\right] + V_{(o)}$$

$$\left[\frac{R_1 + R_3}{R_2}\right] V_{out} + \frac{R_1 R_3}{R_2} C\frac{dV_{out}}{dt} = -V_e - R_3 C\frac{d}{dt}$$

multiply through by $\frac{R_2}{R_1 + R_3}$

$$\Rightarrow V_{out} + \left[\frac{R_2}{R_1 + R_3}\right] R_3 C\frac{dV_{out}}{dt} = -\left[\frac{R_2}{R_1 + R_3}\right] V_e$$
$$- \left[\frac{R_2}{R_1 + R_2}\right] R_3 C\frac{dV_{out}}{dt}$$

inverting

$$V_{out} + \left(\frac{R_1}{R_1 + R_3}\right) R_3 C\frac{dV_{out}}{dt} = \left[\frac{R_2}{R_1 + R_3}\right] V_e + \left(\frac{R_2}{R_1 + R_3}\right) R_3$$

$$V_{out} = \left[\frac{R_2}{R_1 + R_3}\right] V_e + \left(\frac{R_2}{R_1 + R_3}\right) R_3 C\frac{dV_e}{dt} + V_0$$

$$V_{out} = G_p V_e + G_p C_D \frac{dV_e}{dt} + V_0$$

$\Rightarrow G_p = \frac{R_2}{R_1 + R_3}$ – Proportional gain

$G_D = R_3 C$ – Derivative gain