$$C\frac{d}{dt}[V_{out} - V_b] - \frac{V_b}{R_2} = 0 \quad \text{—— (1)}$$

$$\frac{V_e - V_a}{R_2} + C\frac{d}{dt}[V_e - V_b] - \frac{V_a}{R_1} = 0$$

$$\frac{V_{out}}{R_2} + \frac{V_o}{R_1} = 0$$

(2)

PI

$$C\frac{d}{dt}[V_{out} - V_b] - \frac{V_b}{R_2} = 0$$

$$C\frac{d}{dt}[V_{out} - V_b] = \frac{V_b}{R_2}$$

$$\frac{d}{dt}[V_{out} - V_b] = \frac{V_b}{R_2 C}$$

Recall

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0$$

~~$\int d$~~

$$\frac{V_o}{R_2} = \frac{V_e}{R_1}$$

$$V_o = \frac{R_2}{R_1} V_e \quad \text{——}$$

$$\int d\left[V_{out} - V_b\right] = \int \frac{V_b}{R_2C}\, dt$$

$$V_{out} - V_b = \int \frac{V_b}{R_2C}\, dt$$

$$V_{out} = \int \frac{V_b}{R_2C}\, dt + V_b$$

$$= \frac{1}{R_2C} \int -\frac{R_2}{R_1} V_e\, dt + \left[-\frac{R_2 V_c}{R_1}\right]$$

$$= -\frac{1}{R_2C} \frac{R}{R_1} \int_2^t V_e\, dt - \frac{R_2 V_c}{R_1}$$

$$= -\left[\frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot \frac{1}{R_2C} \int_0^t V_e\, dt\right] + V(0)$$

After Inverting

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_e}{R_1} \frac{1}{R_2C} \int_0^t V_e dt + V_{(0)}$$

$$V_{out} = G_p V_e + G_P G_I \int_0^t V_e dt + V_{(0)}$$

Where

$G_P = \frac{R_2}{R_1}$ Proportional Gain

$$G_I = \frac{1}{R_2 C} \quad \text{Integral Gain}$$

PD

$$\frac{V_e - V_o}{R_3} + C\frac{d}{dt}[V_e - V_a] - \frac{V_a}{R_1} = 0 \quad — *$$

$$\frac{V_{out}}{R_2} + \frac{V_o}{R_1} = 0 \quad — **$$

From * *

$$V_a = -\frac{R_1}{R_2} V_{out}$$

Substituting into eqn *

$$\frac{V_e}{R_3} - \left[-\frac{R_1 V_{out}}{R_2}\right] \cdot \frac{1}{R_3} + C\frac{d}{dt} - C\frac{d}{dt}\left[-\frac{R_1 V_{out}}{R_2}\right] - \left[\frac{-R_1 V_{out}}{R_{2}}\right]\frac{1}{R}$$

$$= 0.$$

$$\frac{V_e}{R_3} + \frac{R_1}{R_2 R_3} V_{out} + C\frac{dV_e}{dt} + \frac{R_1}{R_2} C\frac{dV_{out}}{dt} + \frac{1}{R_2} V_{out} = 0$$

Multiply through by R_3

$$V_e + \frac{R_1}{R_2} V_{out} + R_3 C\frac{dV_e}{dt} + \frac{R_1 R_3}{R_2} C\frac{dV_{out}}{dt} + \frac{R_3 V_{out}}{R_2} = 0$$

$$\frac{R_1 V_{out}}{R_2} + \frac{R_1 R_3}{R_2} \frac{C\,d V_{out}}{dt} + \frac{R_3 V_{out}}{R_2} = -V_e - R_3 \frac{C\,dV_e}{dt}$$

$$\frac{R_1 V_{out} + R_3 V_{out}}{R_2} + \frac{R_1 R_3}{R_2} \frac{C\,d V_{out}}{dt} = -V_e - R_3 \frac{C\,d V_e}{dt}$$

$$\frac{R_1 V_{out} + R_3 V_{out}}{R_2} + \frac{R_1 R_3}{R_2} \frac{C\,d V_{out}}{dt} = -V_e - R_3 \frac{C\,d V_e}{dt}$$

$$\left[\frac{R_1 + R_3}{R_2}\right] V_{out} + \frac{R_1 R_3}{R_2} \frac{C\,d V_{out}}{dt} = -V_e - R_3 \frac{C\,d V_e}{dt}$$

Multiply through $\frac{R_2}{R_1+R_3}$

$$V_{out} + \left[\frac{R_1}{R_1+R_3}\right] R_3 \frac{C\,d V_{out}}{dt} = -\left[\frac{R_2}{R_1+R_3}\right] V_e - \left[\frac{R_2}{R_1+R_3}\right] R_3 \frac{C\,d V_e}{dt}$$

After Inverting

$$V_{out} + \left[\frac{R_1}{R_2+R_3}\right] R_3 C \frac{d V_{out}}{dt} = \left(\frac{R_2}{R_1+R_3}\right) V_e + \left(\frac{R_2}{R_1+R_3}\right) R_3 \frac{C\,d V_e}{dt}$$

$$V_{out} = \left(\frac{R_2}{R_1+R_3}\right) V_e + \left(\frac{R_2}{R_1+R_3}\right) R_3 \frac{C\,d V_e}{dt} + V(0)$$

$$V_{out} = G_P V_e + G_P \frac{G_D\,dV_e}{dt} + V_0$$

Where

$$G_P = \frac{R_2}{R_1 + R_3}$$ — Proportional Gain

$$C_D = R_3 C$$ — Derivative gain