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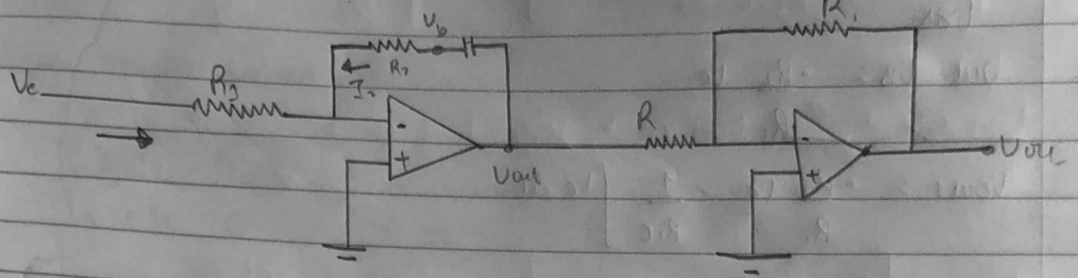
Course Title: Process Control & Automation
Department: Mechatronics

1) Proportional - Integral mode Controller.




Using Kirchoff's Current Law.

$V_a = 0$

$I_1 + I_2 = 0$ ----(i)

$I_2 - I_3 = 0$. ----(ii)

where: [Ohm's Law].

$I_1 = \frac{V_e}{R_1}$

$I_2 = \frac{V_o}{R_2}$

$I_3 = C\frac{dV_c}{dt}$

∴ from equ (i)

$\frac{V_e}{R_1} + \frac{V_o}{R_2}$

∴ from equ (ii)

$\cancel{C\frac{d[V_{out}-V_B]}{dt} - \frac{V_o}{R_2}}$ $C\frac{d[V_{out} - V_B]}{dt} - \frac{V_o}{R_2}$

Combining equation (i) and ii [i.e equ (i) + equ (ii)]

$\frac{V_e}{R_1} + C\frac{d[V_{out} - V_B]}{dt} = 0$.

$C\frac{d[V_{out} - V_B]}{dt} = -\frac{V_e}{R_1}$

Integrating both sides.

$$C[V_{out1} - V_B] = -\frac{1}{R_1}\int V_e\,dt$$

$$V_{out1} = V_B - \frac{1}{R_1 C}\int V_e\,dt$$

~~tom~~
but $V_B = -\frac{R_2}{R_1} V_e$.

$$V_{out1} = -\frac{R_2}{R_1} V_e - \frac{1}{R_1 C}\int V_e\,dt$$

$$V_{out1} = -\frac{R_2}{R_1} V_e - \frac{R_2}{R_1} \cdot \frac{1}{R_2 C}\int V_e\,dt.$$

i.e $\frac{R_2}{R_2} = 1$.

After inverting
$-V_{out1} = V_{out}$..

∴

$$-V_{out1} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot \frac{1}{R_2 C}\int_0^t V_e\,dt$$ [i.e multiplying both sides

becomes

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot \frac{1}{R_2 C}\int_0^t V_e\,dt + V(0)$$

Considering initial conditions.

Or.

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e\,dt + V(0).$$

where

$G_P = \frac{R_2}{R_1}$ -

$G_I = \frac{1}{R_2 C}$ — Integral gain.

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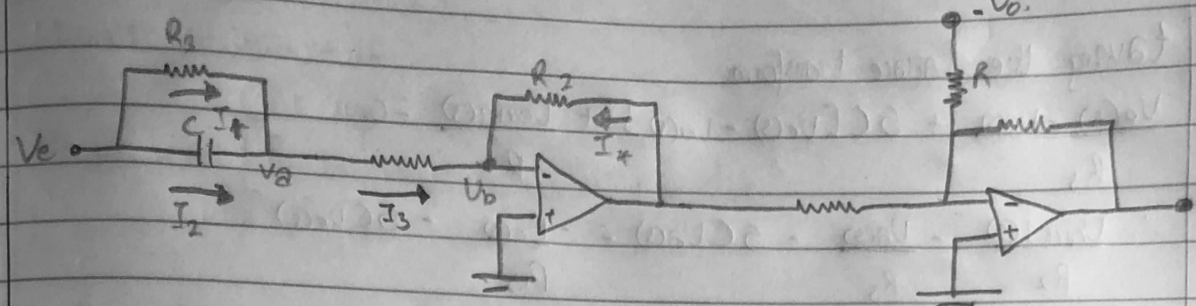


Considering $2\pi f_{max} RC = 0$.

Using Kirchoff's Current law.

$$I_1 + I_2 - I_3 = 0 \quad \text{---(i)}$$
$$I_x + I_3 = 0. \quad \text{---(ii)}$$

$$R = \frac{R_1 R_3}{R_1 + R_3}$$

Where R is the effective resistance

As per Ohm's law:

$$I_1 = \frac{V_e - V_a}{R_3}$$

$$I_2 = C \frac{d}{dt}[V_e - V_a]$$

$$I_3 = \frac{V_a}{R_1}$$

$$I_x = \frac{V_{out1}}{R_2}.$$

∴ from equ (i).

$$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}[V_e - V_a] - \frac{V_a}{R_1} = 0.$$

from equ (ii)

$$\frac{V_{out1}}{R_2} + \frac{V_a}{R_1} = 0.$$

Summing equ(i) and equ(ii)

$$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}[V_e - V_a] + \frac{V_{out1}}{R_2} = 0.$$

taking the Laplace transform.

$$\frac{V_e(s) - V_a(s)}{R_3} + SC[V_e(s) - V_a(s)] + \frac{V_{out1}(s)}{R_2} = 0.$$

$$\therefore \frac{V_{out1}(s)}{R_2} - \frac{V_a(s)}{R_3} - SC(V_a(s)) = -\frac{V_e(s)}{R_3} - SC(V_e(s))$$

Expressing $V_a(s)$ in terms of $V_{out}$.

$$\frac{V_{out1}(s)}{R_2} - \frac{1}{R_3}\left[-\frac{R_1}{R_2}V_{out1}(s)\right] - SC\left[-\frac{R_1}{R_2}V_{out1}(s)\right] = -\frac{V_e(s)}{R_3} - S(V_e(s))$$

$$\frac{V_{out1}(s)}{R_2} + \frac{R_1}{R_2R_3}V_{out1}(s) + \frac{SCR_1}{R_2}V_{out1}(s) = -\frac{V_e(s)}{R_3} - SC(V_e(s))$$

$$V_{out1}(s)\left[\frac{1}{R_2} + \frac{R_1}{R_2R_3} + \frac{SCR_1}{R_2}\right] = -\frac{V_e(s)}{R_3} - SC(V_e(s))$$

$$V_{out1}(s)\left[\frac{R_3 + \overset{R_1}{\cancel{R_2R_3}} + SCR_1R_3}{R_2R_3}\right] = -\frac{V_e(s)}{R_3} - SC(V_e(s))$$

$$V_{out1}(s) = \left[\frac{R_2R_3}{R_3 + \overset{R_1}{\cancel{R_2R_3}} + SCR_1R_3}\right]\left[-\frac{V_e(s)}{R_3} - SC(V_e(s))\right]$$

$$V_{out1}(s) = \left[\frac{R_2R_3/R_1+R_3}{R_1+R_3/R_1+R_3 + SCR_1R_3/R_1+R_3}\right]\left[-\frac{V_e(s)}{R_3} - SC(V_e(s))\right]$$

$$V_{out1}(s) = \left[\frac{\frac{R_2R_3}{R_1+R_3}}{1+SCR}\right]\left[-\frac{V_e(s)}{R_3} - SC(V_e(s))\right]$$

if $SCR << 1$.

$$V_{out1} = \left[\frac{R_2R_3}{R_1+R_3}\right]\left[-\frac{V_e(s)}{R_3} - SC(V_e(s))\right]$$

$$\therefore V_{out1}(s) = -\left[\frac{R_2}{R_1+R_3}\right]V_e(s) - \left[\frac{R_2R_3}{R_1+R_3}\right]SC(V_e(s))$$

taking the inverse laplace transform.

$$V_{out1} = -\left[\frac{R_2}{R_1+R_3}\right]V_e - \left[\frac{R_2R_3}{R_1+R_3}\right]C\frac{dV_c}{dt}$$

from the inverter amp.

$$V_{out1} = -(V_{out} + (-V_o)]$$

$V_{out}\,t = -V_{out} + V_o$

$$V_{out}\,i = -\left[\frac{R_2}{R_1+R_3}\right]V_e - \left[\frac{R_2R_3}{R_1+R_3}\right]C\frac{dV_e}{dt}$$

$$\therefore -V_{out} + V_o = -\left[\frac{R_2}{R_1+R_3}\right]V_e - \left[\frac{R_2R_3}{R_1+R_3}\right]C\frac{dV_e}{dt}$$

$$-V_{out} = -\left[\frac{R_2}{R_1+R_3}\right]V_e - \left[\frac{R_2R_3}{R_1+R_3}\right]C\frac{dV_e}{dt} - V_o$$

$\therefore$

$$V_{out} = \left[\frac{R_2}{R_1+R_3}\right]V_e + \left[\frac{\cancel{R_2R_3}\;R_2}{R_1+R_3}\right]R_3C\frac{dV_e}{dt} + V_o$$

Where

$G_P = \frac{R_2}{R_1+R_3}$ --- Proportional Gain

$G_D = R_3C$ --- Derivative Gain

$$\therefore V_{out} = G_PV_e + G_PG_D\frac{dV_e}{dt} + V_o$$

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