MCT 511 ASSIGNMENT

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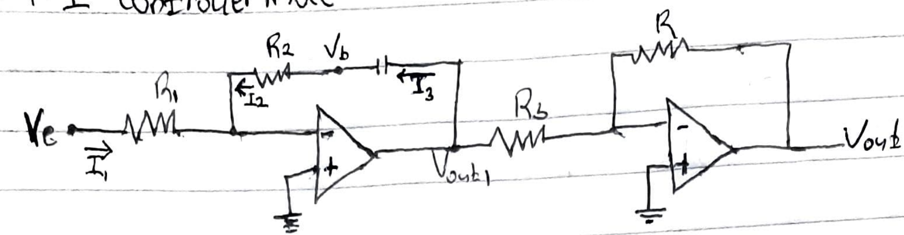
① P.I Controller mode



No current & Voltage exists btw the OP-Amplifier Input terminal

$\therefore \quad V_0 = 0$

$I_1 + I_2 = 0$

$I_3 - I_2 = 0$

Relationship btw Voltage & Current across the Capacitor:

$I_c = C\frac{dV_c}{dt}$

$I_1 = \frac{V_e - V_a}{R_1}, \quad V_a = 0$

$I_1 = \frac{V_e}{R_1}$

$I_2 = \frac{V_b - V_a}{R_2}, \quad V_a = 0$

$I_2 = \frac{V_b}{R_2}$

$I_1 + I_2 = \frac{V_e}{R_1} + \frac{V_b}{R_2} \quad -- ①$

$I_3 = I_c, \quad I_3 = C\frac{dV_c}{dt}, \quad I_2 = \frac{V_b}{R_2}$

$I_3 - I_2 = 0$

$C\frac{dV_c}{dt} - \frac{V_b}{R_2} = 0$

$C\frac{d[V_{out1} - V_b]}{dt} - \frac{V_b}{R_2} = 0 \quad -- ②$

From eqn ①

$\frac{V_b}{R_2} = \frac{-V_e}{R_1}$

Making V_b subject of formula

$V_b = \frac{-V_e R_2}{R_1} \quad -- ③$

Taking Laplace transform of eqn 2

$$SC[V_{out,1}(s) - V_b(s)] - \frac{V_b(s)}{R_2} = 0$$

$$SC[V_{out,1}(s) - V_b(s)] = \frac{V_b(s)}{R_2}$$

$$SCV_{out,1}(s) - SCV_b(s) = \frac{V_b(s)}{R_2}$$

$$SCV_{out,1}(s) = \frac{V_b(s)}{R_2} + SCV_b(s)$$

$$SCV_{out,1}(s) = V_b(s)\left(\frac{1}{R_2} + SC\right) \quad \text{-- } *$$

Substituting eqn 3 into eqn *

$$SCV_{out,1}(s) = -\frac{V_e(s)R_2}{R_1}\left(\frac{1}{R_2} + SC\right)$$

$$SCV_{out,1}(s) = -\frac{V_e(s)}{R_1} - \frac{V_e(s)SCR_2}{R_1}$$

$$V_{out,1}(s) = -\frac{V_e(s)}{R_1SC} - \frac{V_e(s)R_2}{R_1}$$

From the inverter side of the circuit

$$V_{out,1} = -V_{out}$$

$$V_{out}(s) = -\left(-\frac{V_e(s)}{R_1SC} - \frac{V_e(s)R_2}{R_1}\right)$$

$$V_{out}(s) = \frac{V_e(s)}{SCR_1} + \frac{V_e(s)R_2}{R_1}$$

Taking inverse Laplace

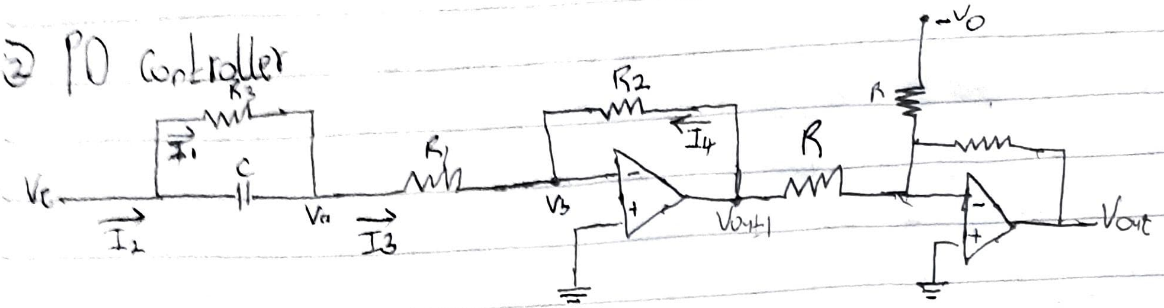
$$\cancel{V_{out}(s) = R_2} \quad V_{out} = \frac{V_e(t)R_2}{R_1} + \frac{R_2}{R_1}\cdot\frac{1}{R_2}\int_0^t -V_e\,dt + V_i$$

$$V_{out} = G_pV_e(t) + G_pG_I\int_0^t V_e\,dt + V_0$$

$$G_p = R_2/R_1$$

$$G_I = 1/R_2C$$

② PD Controller



$$I_1 + I_2 = I_3 \quad \text{--- ①}$$

$$I_4 + I_3 = 0 \quad \text{--- ②}$$

$$R = \frac{R_1 R_3}{R_1 + R_3} \rightarrow \text{Effective resistance}$$

$$I_1 = \frac{V_e - V_a}{R_3}$$

$$I_2 = C\frac{d}{dt}(V_e - V_a)$$

$$I_3 = \frac{V_a - V_b}{R_1}, \quad V_b = 0$$

$$I_3 = \frac{V_a}{R_1}$$

$$I_4 = \frac{V_{out1} - V_b}{R_2}, \quad V_b = 0$$

$$I_4 = \frac{V_{out1}}{R_2}$$

$$I_1 + I_2 = I_3 \rightarrow \frac{V_e - V_a}{R_3} + C\frac{d}{dt}(V_e - V_a) = \frac{V_a}{R_1} \quad \text{--- \#}$$

$$I_4 + I_3 = 0 \rightarrow \frac{V_a}{R_1} + \frac{V_{out1}}{R_2} = 0 \quad \text{--- \#\#}$$

From eqn ## $\quad \frac{V_a}{R_1} = -\frac{V_{out1}}{R_2}$

Making V_a subject of formula

$$V_a = -\frac{R_1 V_{out1}}{R_2} \quad \text{---- *}$$

~~From Substitute eqn * into eqn #~~

From eqn # $\quad \frac{V_e - V_a}{R_3} + C\frac{d}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0 \quad \text{--- **}$

Substituting eqn * into **

$$\frac{V_e}{R_3} - \left[\frac{-R_1 V_{out1}}{R_2}\right]\frac{1}{R_3} + C\frac{dV_e}{dt} - C\frac{d}{dt}\left[\frac{-R_1 V_{out1}}{R_2}\right] - \left[\frac{-R_1}{R_2} V_{out1}\right]\frac{1}{R_1} = 0$$

$$\frac{V_e}{R_3} + \frac{R_1 V_{out1}}{R_2 R_3} + C\frac{dV_e}{dt} + \frac{R_1}{R_2} C\frac{d V_{out1}}{dt} + \frac{1}{R_2} V_{out1} = 0$$

Multiplying through by R_3

$$V_e + \frac{R_1}{R_2} V_{out1} + R_3 C\frac{dV_e}{dt} + \frac{R_1 R_3}{R_2} C\frac{dV_{out1}}{dt} + \frac{R_3}{R_2} V_{out} = 0$$

$$\frac{R_1}{R_2} V_{out1} + \frac{R_3}{R_2} V_{out1} + \frac{R_1 R_3}{R_2} C\frac{dV_{out1}}{dt} = -V_e - R_3 C\frac{dV_e}{dt}$$

$$\frac{R_1 V_{out1} + R_3 V_{out1}}{R_2} + \frac{R_1 R_3}{R_2} C\frac{d V_{out1}}{dt} = -V_e - R_3 C\frac{dV_e}{dt}$$

$$\cancel{\frac{R_1+R_3}{R_2}}\left[\frac{R_1+R_3}{R_2}\right] V_{out1} + \frac{R_1 R_3}{R_2} C\frac{d V_{out1}}{dt} = -V_e - R_3 C\frac{d V_e}{dt}$$

Multiplying through by $\frac{R_2}{R_1+R_3}$

$$V_{out} + \left[\frac{R_1 R_3}{R_1+R_3}\right] C\frac{d V_{out}}{dt} = \left[\frac{R_2}{R_1+R_3}\right] V_e - \left[\frac{R_2}{R_1+R_3}\right] R_3 C\frac{dV_e}{dt}$$

From the inverter side of the circuit

$$V_{out1} = -V_{out}$$

$$V_{out} + \left[\frac{R_1 R_3}{R_2+R_3}\right] C\frac{d V_{out}}{dt} = \left[\frac{R_2}{R_1+R_3}\right] V_e + \left[\frac{R_2}{R_1+R_3}\right] R_3 C\frac{d V_e}{dt}$$

$$V_{out} = \left[\frac{R_1}{R_1+R_3}\right] V_e + \left[\frac{R_2}{R_1+R_3}\right] R_3 C\frac{dV_e}{dt} + V_0$$

$$V_{out} = G_p V_e + G_p G_D \frac{dV_e}{dt} + V_0$$

$G_p = \frac{R_2}{R_1+R_3}$ → proportional gain

$G_D = R_3 C$ ⟶ Derivative gain