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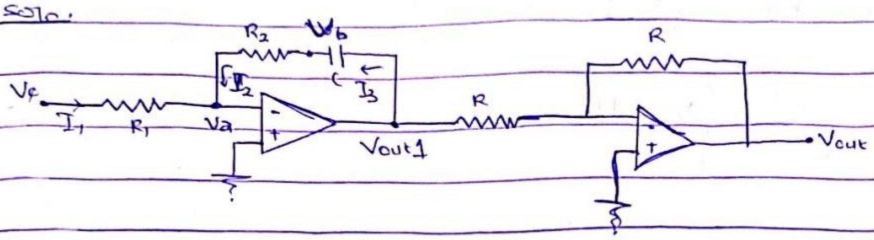
16/ENG04/043 - Electrical Electronics Engineering

EEE 561 - Process control & Automation

Question

Derive the analysis for the output voltage using operational amplifier for proportional integral controller mode.

Solu:



Since no current flows through the op-amp, $V_a = 0$ (no voltage across its terminals)

* KCL $\quad I_3 = I_2 \;;\; I_3 - I_2 = 0$

$$I_1 + I_2 = 0$$

$$I_3 - I_2 = 0$$

recall $I = C\frac{dv}{dt}$ hence I_3 flowing through capacitor C =

$$I_3 = C\frac{d(V_{out_1} - V_b)}{dt}$$

$$I_1 = \frac{V_e - V_a}{R_1} = \frac{V_e - 0}{R_1} = \frac{V_e}{R_1}$$

$$I_2 = \frac{V_b - V_a}{R_2} = \frac{V_b - 0}{R_2} = \frac{V_b}{R_2}$$

substituting the value into KCL equations above.

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \;;\; \frac{V_b}{R_2} = -\frac{V_e}{R_1} \;;\; V_b = -\frac{R_2}{R_1} \cdot V_e \quad \text{--- (eqn*)}$$

$$C\frac{d(V_{out_1} - V_b)}{dt} - \frac{V_b}{R_2} = 0$$

recall $\frac{V_b}{R_2} = -\frac{V_e}{R_1}$

$$\cancel{C\frac{d(V_{out} - V_b)}{dt} + \frac{V_e}{R_1} = 0}$$

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*can also be done using laplace!!

$$C\frac{d(V_{out_1} - V_b)}{dt} - \frac{V_b}{R_2} = 0$$

recall $V_b = -\frac{R_2}{R_1} \cdot V_e$

$$C\frac{d}{dt}(V_{out_1} - V_b) + \frac{R_2}{R_1} \cdot \frac{1}{R_2} \cdot V_e = 0$$

$$\frac{d}{dt} V_{out_1} - V_b = \frac{-1}{C} \cdot \frac{R_2}{R_1} \cdot \frac{1}{R_2} \cdot V_e$$

$$d(V_{out_1} - V_b) = \frac{-R_2}{R_1 R_2 \cdot C} \cdot V_e \cdot dt$$

Take integral of both sides.

$$\int d(V_{out_1} - V_b) = \int \frac{-R_2}{R_1 R_2 \cdot C} \cdot V_e \, dt$$

$$V_{out_1} - V_b = \frac{-R_2}{R_1 R_2 C} \cdot \int V_e \, dt$$

$$V_b = -\frac{R_2}{R_1} V_e$$

$$V_{out_1} + \frac{R_2}{R_1} V_e = -\frac{R_2}{R_1} \cdot \frac{1}{R_2 \cdot C} \int V_e \, dt$$

$$V_{out} = -\frac{R_2}{R_1} V_e - \frac{R_2}{R_1} \cdot \frac{1}{R_2 \cdot C} \int V_e \cdot dt$$

inverting circuit $V_{out} = -\frac{R}{R}(V_{out}) = -V_{out}$ hence $V_{out} = -V_{out}$

inverting

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot \frac{1}{R_2 C} \int V_e \cdot dt$$

Taking initial condition into consideration.

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} \cdot \frac{1}{R_2 \cdot C} \int V_e \cdot dt + V_{(0)}$$

recall for PI controller mode

$\cancel{P = K_p e_p + K_p K_I \int e_p \, dt + P_I(0)}$

$$V_{out} = G_p V_e + G_p G_I \int_0^t V_e \, dt + V_{(0)} \quad \text{------} \quad (eqn^{***})$$

comparing coefficients,

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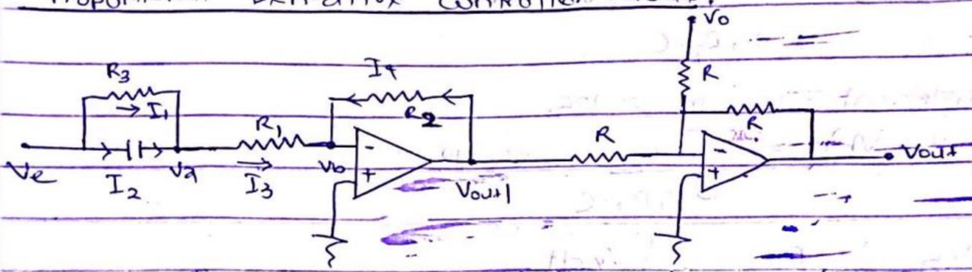
$$G_P = \frac{R_2}{R_1} = \text{proportional gain}$$

$$G_I = \frac{1}{R_2 \cdot C} = \text{integral gain}$$

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e \cdot dt + V_{(0)}$$

Question 2.

Proportional Derivative controller mode.



No current flows through the op-amp, hence no voltage across)

$V_b = 0$.

finding equation for I using KCL.

$I_1 + I_2 = I_3$ ... $I_1 + I_2 - I_3 = 0$

$I_3 + I_4 = 0$ $\qquad$ $I_1 + I_2 + I_4 = 0$

Resolving parallel resistors R_1 & R_3

$R = \frac{R_1 \times R_3}{R_1 + R_3}$; R = effective resistance

$2\pi f_{max} RC = 0.1$.

finding I using ohms law.

$$I_1 = \frac{V_e - V_a}{R_3}$$

$$I_2 = \frac{C\,d(V_e - V_a)}{dt}$$

$$I_3 = \frac{V_a - V_b}{R_1} = \frac{V_a}{R_1}$$

$$I_4 = \frac{V_{out_1} - V_b}{R_2} = \frac{V_{out_1}}{R_2}$$

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Substituting I into KCL eqn.

$$\frac{V_\phi - V_a}{R_3} + C\frac{d(\cancel{V_{out_1}}\, V_\phi - V_a)}{dt} - \frac{V_a}{R_1} = 0$$

$$\frac{V_{out} - V_b}{R_2} + \frac{V_a - V_b}{R_1} = I_3 + I_4 = 0 \quad ; V_b = 0$$

$$\frac{V_{out_1}}{R_2} + \frac{V_a}{R_1} = 0$$

$$V_a = -\frac{R_1}{R_2} \cdot V_{out_1}$$

$$I_1 + I_2 + I_4 = 0$$

$$\frac{V_\phi - V_a}{R_3} + C\frac{d(V_\phi - V_a)}{dt} + \frac{V_{out_1} + V}{R_2} = 0$$

~~$V_\phi - V_a$~~ Taking laplace transform.

$$\frac{V_\phi(s) - V_a(s)}{R_3} + sC[V_\phi(s) - V_a(s)] + \frac{V_{out_1}(s)}{R_2} = 0$$

$$\frac{V_\phi(s)}{R_3} + sC\,V_\phi(s) + \frac{V_{out_1}(s)}{R_2} = \frac{V_a(s)}{R_3} + sCV_a(s)$$

$$\frac{V_{out}(s)}{R_2} - \frac{V_a(s)}{R_3} - sCV_a(s) = -\frac{V_\phi}{R_3} - sCV_\phi(s)$$

recall $V_a = -\frac{R_1}{R_2} \cdot V_{out}$

$$\frac{V_{out}(s)}{R_2} + \frac{R_1}{R_2} \cdot \frac{1}{R_3} \cdot V_{out} + sC \cdot \frac{R_1}{R_2} \cdot V_{out} = -\frac{V_\phi(s)}{R_3} - sCV_\phi(s)$$

$$V_{out}(s)\left[\frac{1}{R_2} + \frac{R_1}{R_2 R_3} + \frac{sCR_1}{R_2}\right] = -\frac{V_\phi(s)}{R_3} - sCV_\phi(s)$$

$$V_{out}(s)\left[\frac{R_3 + R_1 + sCR_1R_3}{R_2R_3}\right] = -\frac{V_\phi(s)}{R_3} - sCV_\phi(s)$$

$$V_{out}(s) = \left[\frac{R_2R_3}{R_3 + R_1 + sCR_1R_3}\right] \cdot -\frac{V_\phi(s)}{R_3} - sCV_\phi(s)$$

$$V_{out}(s) = \left[\frac{(R_2R_3)/R_1 + R_3}{R_3 + R_1 / R_1 + R_3 + \frac{sCR_1R_3}{R_1 + R_3}}\right] \cdot -\frac{V_\phi(s)}{R_3} - sCV_\phi(s)$$

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$$V_{out}(s) = \left[\frac{R_2R_3 / R_1+R_3}{1 + SC\frac{R_1R_3}{R_1+R_3}}\right] \cdot \left[\frac{-V_e(s)}{R_3} - SCV_e(s)\right]$$

recall R = effective resistance $= \frac{R_1R_3}{R_1+R_3}$

$$V_{out}(s) = \left[\frac{R_2R_3 / R_1+R_3}{1 + SCR}\right]\left[\frac{-V_e(s)}{R_3} - SCV_e(s)\right]$$

if $SCR << 1$ $\quad 1+SCR \simeq 1$

$$V_{out}(s) = \frac{R_2R_3}{R_1+R_3}\left[\frac{-V_e(s)}{R_3} - SCV_e(s)\right]$$

$$V_{out}(s) = -V_e(s) \cdot \left[\frac{R_2}{R_1+R_3}\right] - SCV_e(s)\left[\frac{R_2R_3}{R_1+R_3}\right]$$

inverse laplace L^{-1}

$$V_{out,1}(t) = -V_e\left[\frac{R_2}{R_1+R_3}\right] - C\frac{dV_e}{dt} \cdot \left[\frac{R_2R_3}{R_1+R_3}\right]$$

from the inverter circuit,

$$V_{out,1} = -(V_{out} - V_0) = V_0 - V_{out}$$

$$V_0 - V_{out} = -V_e\left[\frac{R_2}{R_1+R_3}\right] - C\frac{dV_e}{dt}\left[\frac{R_2R_3}{R_1+R_3}\right]$$

$$-V_{out} = -\left[\frac{R_2}{R_1+R_3}\right] \cdot V_e - \left[\frac{R_2R_3}{R_1+R_3}\right] \cdot C\frac{dV_e}{dt} - V_0$$

$$V_{out} = V_e\left[\frac{R_2}{R_1+R_3}\right] + R_3C\frac{dV_e}{dt}\left[\frac{R_2}{R_1+R_3}\right] + V_0$$

proportional gain $G_P = \frac{R_2}{R_1+R_3}$

~~der~~ derivative gain $G_D = R_3C$

$$V_{out} = G_P \cdot V_e + G_D G_P \cdot \frac{dV_e}{dt} + V_0$$

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