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16/ENG04/014
EEE561 Assignment

Question
Derive the analysis for the output voltage in using Operational amplifier for
i) Proportional Integral Controller mode
ii) Proportional derivative mode

— Proportional Integral Controller mode
$V_a = 0$
$I_1 + I_2 = 0$ - - - (i)
$I_3 - I_2 = 0$ - - - (ii)

Current flowing through the capacitor
$$I_c = \frac{c\,dV_c}{dt}$$
$$I_1 = \frac{V_e - V_a}{R_1}$$
recall $V_a = 0$
$$I_1 = \frac{V_e - 0}{R_1} = \frac{V_e}{R_1}$$

$$I_2 = \frac{V_b - V_a}{R_1} = \frac{V_b - 0}{R_2}$$
$$I_2 = \frac{V_b}{R_2}$$

$$I_3 = \frac{Cd}{dt}\left(V_{out_1} - V_b\right)$$

Sub into eqn (i) $ Eqn (ii)

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \quad --- (i)$$

$$C\frac{d}{dt}\left(V_{out1} - V_b\right) - \frac{V_b}{R_2} = 0 \quad -- (ii)$$

from eq (i)

$$\frac{V_b}{R_2} = -\frac{V_e}{R_1}$$

$$V_b = -\frac{R_2 V_e}{R_1}$$

$$V_b = \frac{-R_2 V_e}{R_1}$$

taking the laplace transform of eq (ii)

$$SC\left(V_{out1}(s) - V_b(s)\right) - \frac{V_b(s)}{R_2} = 0$$

$$SC\, V_{out1}(s) = SC\, V_b(s) + \frac{V_b(s)}{R_2}$$

$$SC\, V_{out1}(s) = V_b(s)\left(SC + \frac{1}{R_2}\right)$$

recall; $V_b = \frac{-R_2}{R_1} V_e$

$$SC\, V_{out1}(s) = -\frac{R_2}{R_1} V_e(s)\left(SC + \frac{1}{R_2}\right)$$

$$V_{out,1}(s) = -\frac{R_2}{SCR_1} V_e(s) + \frac{V_b(s)}{R_2} - \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)$$

from the Inverting circuit

$$V_{out}(i) = -V_{out}$$

$$V_{out}(s) = -\left[-\frac{R_2}{R_1} V_e(s) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)\right]$$

$$V_{out}(s) = \left[\frac{R_2}{R_1}(V_e(s)) - \frac{R_2}{R_1}\frac{1}{SCR_2} V_e(s)\right]$$

taking the Inverse laplace

$$V_{out} = \frac{R_2}{R_1} V_e(s) + \frac{R_2}{R_1}\frac{1}{R_2}\int_0^t V_e(t)\,dt + V(0)$$

$$\text{Where } \frac{1}{S} = \int_0^t dt + k$$

$$V_{out} = G_P V_e + G_P G_I \int_0^t V_e\,dt + V(0)$$

where

$$G_P = \frac{R_2}{R_1}$$

$$G_I = \frac{1}{R_2 C}$$

ii) Proportional Derivative Controller mode

$I_1 + I_2 = I_3$ --(i)

$I_3 + I_4 = 0$ --(ii)

$I_1 = \dfrac{V_e - V_a}{R_3}$

$I_2 = \dfrac{Cd}{dt}(V_e - V_a)$

$I_3 = \dfrac{V_a - V_b}{R_1}$ $(V_b = 0)$

$\dfrac{V_a}{R_1}$

$I_4 = \dfrac{V_{out1} - V_b}{R_2}$

$V_b = 0$

$I_4 = \dfrac{V_{out1}}{R_2}$

$R = \dfrac{R_1 R_3}{R_1 + R_3}$ ---- effective resistance

Sub into eqn (i) & (ii)

$\dfrac{V_e - V_a}{R_3} + \dfrac{Cd}{dt}(V_e - V_a) = \dfrac{V_a}{R_1}$ ------ (1)

$\dfrac{V_a}{R_1} + \dfrac{V_{out1}}{R_2} = 0$ --- (2)

from eqn (2)

$\dfrac{V_a}{R_1} = -\dfrac{V_{out1}}{R_2}$

$$V_a = -\frac{R_1}{R_2} V_{out1}$$

rearranging eq (1)

$$\frac{V_e - V_a}{R_3} + \frac{Cd}{dt}(V_e - V_a) - \frac{V_a}{R_1} = 0$$

taking laplace transform

$$\frac{V_{e(s)} - V_{a(s)}}{R_3} + sC\left(V_{e(s)} - V_{a(s)}\right) - \frac{V_{a(s)}}{R_1} = 0$$

As the initial conditions turn to zero

$$\frac{V_{e(s)}}{R_3} + sCV_{e(s)} = \frac{V_{a(s)}}{R_1} + \frac{V_{a(s)}}{R_3} + sC V_{a(s)}$$

$$V_{a(s)}\left[\frac{1}{R_3} + sC\right] = V_{a(s)}\left[\frac{1}{R_1} + \frac{1}{R_3} + sC\right]$$

recall $V_a = -\frac{R_1}{R_2} V_{out1}$

$$V_{e(s)}\left[\frac{1}{R_3} + sC\right] = -\frac{R_1}{R_2} V_{out1(s)}\left[\frac{1}{R_1} + \frac{1}{R_3} + sC\right]$$

taking the Lcm

$$V_{e(s)}\left[\frac{1 + R_3 sC}{R_3}\right] = -\frac{R_1}{R_3} V_{out1(s)}\left[\frac{R_3 + R_1 + sCR_1R_3}{R_1 R_3}\right]$$

$$V_{e(s)}\left(1 + sCR_3\right) = -\frac{V_{out1(s)}}{R_2}\left(R_3 + R_1 + sCR_1R_3\right)$$

$$-V_{out1(s)} = \frac{V_{e(s)}\left(1 + sCR_3\right)R_2}{\left(R_1 + R_3 + sCR_1R_3\right)}$$

$$-V_{out1}(s) = \frac{V_{e(s)}\left(R_2 + SCR_2R_3\right)}{\left(R_1 + R_3 + SCR_1R_3\right)}$$

Dividing through by $R_1 + R_3$

$$-V_{out1}(s) = \frac{V_{e(s)}\left(R_2 + SCR_2R_3\right) / R_1 + R_3}{\frac{R_1 + R_3}{R_1 + R_3} + \frac{SCR_1\cancel{R}R_3}{R_1 + R_3}}$$

recall that

$$R = \frac{R_1R_3}{R_1 + R_3}$$

$$-V_{out1}(s) = \frac{V_{e(s)}\left(R_2 + S(R_2R_3)\right) / R_1 + R_3}{1 + SCR}$$

If $SCR << 1$

$$-V_{out1}(s) = \frac{V_{e(s)}\left(R_2 + SCR_2R_3\right)}{R_1 + R_3}$$

from the Inverting circuit

$$V_{out1} = -V_{out} + V_0$$

$$\therefore -\left(-V_{out(s)} + V_{(0)}\right) = \frac{V_e\left(R_2 + SCR_2R_3\right)}{R_1 + R_3}$$

$$V_{out(s)} - V_{(0)} = \frac{V_{e(s)}R_2}{R_1 + R_2} + \frac{SCR_2R_3 V_{e(s)}}{R_1 + R_3}$$

$$V_{out(s)} = \frac{R_2}{R_1 + R_3} V_{e(s)} + \frac{R_2}{R_1 + R_3} R_3 C \left(s V_{e(s)}\right) + V_{(0)}$$

taking the Inverse laplace

$$V_{out} = \frac{R_2}{R_1 + R_3} V_e + \frac{R_2}{R_1 + R_3} R_3 C \frac{dV_e}{dt} + V_{(0)}$$

$$V_{out}(s) = \frac{R_2}{R_1+R_3} V_e(s) + \frac{R_2}{R_1+R_3} R_3 C s V_e(s) + V(0)$$

taking Inverse laplace

$$V_{out} = \frac{R_2}{R_1+R_3} V_e + \frac{R_2}{R_1+R_3} R_3 C \frac{dV_e}{dt} + V_0$$

$$V_{out} = G_P V_e + G_P G_D \frac{dV_e}{dt} + V_0$$

where

$$G_P = \frac{R_2}{R_1+R_3}$$

$$G_D = R_3 C$$